10 CV 35 FLUID MECHANICS NOTES

UNIT-1 BASIC PROPOERTIES OF FLUIDS

by

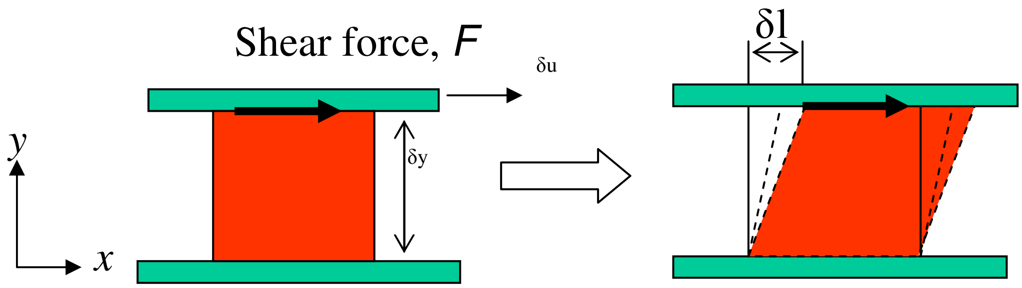
Dr. Nagaraj Sitaram, Professor, Civil Department, SBMJCE, Bangalore

UNIT –1: BASIC PROPERTIES OF FLUIDS:

1.0 INTRODUCTION: In general matter can be distinguished by the physical forms known as solid, liquid, and gas. The liquid and gaseous phases are usually combined and given a common name of fluid. Solids differ from fluids on account of their molecular structure (spacing of molecules and ease with which they can move). The intermolecular forces are large in a solid, smaller in a liquid and extremely small in gas.

Fluid mechanics is the study of fluids at rest or in motion. It has traditionally been applied in such area as the design of pumps, compressor, design of dam and canal, design of piping and ducting in chemical plants, the aerodynamics of airplanes and automobiles. In recent years fluid mechanics is truly a 'high-tech' discipline and many exciting areas have been developed like the aerodynamics of multistory buildings, fluid mechanics of atmosphere, sports, and micro fluids.

1.1 DEFINITION OF FLUID: A *fluid* is a substance which deforms continuously under the action of shearing forces, however small they may be. Conversely, it follows that: If a fluid is at rest, there can be no shearing forces acting and, therefore, all forces in the fluid must be perpendicular to the planes upon which they act.



Fluid deforms continuously under the action of a shear force

$$\tau_{yx} = \frac{dF_x}{dA_y} = f(\text{Deformation Rate})$$

Shear stress in a moving fluid:

Although there can be no shear stress in a fluid at rest, shear stresses are developed when the fluid is in motion, if the particles of the fluid move relative to each other so that they have different velocities, causing the original shape of the fluid to become distorted. If, on the other hand, the velocity of the fluid is same at every point, no shear stresses will be produced, since the fluid particles are at rest relative to each other.

Differences between solids and fluids: The differences between the behaviour of solids and fluids under an applied force are as follows:

i. For a solid, the strain is a function of the applied stress, providing that the elastic limit is not exceeded. For a fluid, the rate of strain is proportional to the applied stress.
ii. The strain in a solid is independent of the time over which the force is applied and, if the elastic limit is not exceeded, the deformation disappears when the force is removed. A fluid continues to flow as long as the force is applied and will not recover its original form when the force is removed.

Differences between liquids and gases:

Although liquids and gases both share the common characteristics of fluids, they have many distinctive characteristics of their own. A liquid is difficult to compress and, for many purposes, may be regarded as incompressible. A given mass of liquid occupies a fixed volume, irrespective of the size or shape of its container, and a free surface is formed if the volume of the container is greater than that of the liquid.

A gas is comparatively easy to compress (Fig.1). Changes of volume with pressure are large, cannot normally be neglected and are related to changes of temperature. A given mass of gas has no fixed volume and will expand continuously unless restrained by a containing vessel. It will completely fill any vessel in which it is placed and, therefore, does not form a free surface.

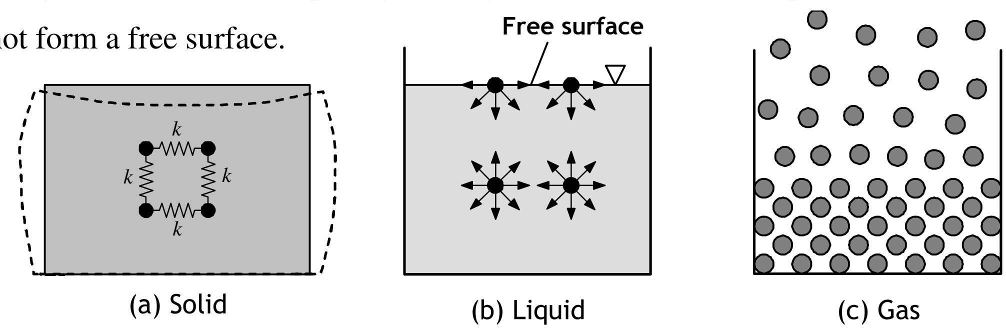


Fig.1 Comparison of Solid, Liquid and Gas

1.2 Systems of Units:

The official international system of units (System International Units). Strong efforts are underway for its universal adoption as the exclusive system for all engineering and science, but older systems, particularly the cgs and fps engineering gravitational systems are still in use and probably will be around for some time. The chemical engineer finds many physiochemical data given in cgs units; that many calculations are most conveniently made in fps units; and that SI units are increasingly encountered in science and engineering. Thus it becomes necessary to be expert in the use of all three systems.

SI system:

Primary quantities:

Quantity	***Unit***
Mass in Kilogram	kg
Length in Meter	m
Time in Second	s or as sec
Temperature in Kelvin	K
Mole	mol

Derived quantities:

Quantity	***Unit***
Force in Newton (1 N = 1 kg.m/s^2)	N
Pressure in Pascal (1 Pa = 1 N/m^2)	N/m^2
Work, energy in Joule (1 J = 1 N.m)	J
Power in Watt (1 W = 1 J/s)	W

CGS Units:
The older centimeter-gram-second (cgs) system has the following units for derived quantities:

Quantity	***Unit***
Force in dyne (1 dyn = 1 g.cm/s^2)	dyn
Work, energy in erg (1 erg = 1 dyn.cm = 1 x 10^{-7} J)	erg
Heat Energy in calorie (1 cal = 4.184 J)	cal

Dimensions: Dimensions of the primary quantities:

Fundamental dimension	***Symbol***
Length	L
Mass	M
Time	t
Temperature	T

Dimensions of derived quantities can be expressed in terms of the fundamental dimensions.

Quantity	*Representative symbol*	*Dimensions*
Angular velocity	ω	t^{-1}
Area	A	L^2
Density	ρ	M/L^3
Force	F	ML/t^2
Kinematic viscosity	ν	L^2/t
Linear velocity	v	L/t

1.3 Properties of fluids:

1.3.1 Mass density or Specific mass (ρ):

Mass density or specific mass is the mass per unit volume of the fluid.

$$\therefore \quad \rho = \frac{Mass}{Volume}$$

$$\rho = \frac{M}{V} \text{ or } \frac{dM}{dV}$$

Unit: kg/m^3

With the increase in temperature volume of fluid increases and hence mass density decreases in case of fluids as the pressure increases volume decreases and hence mass density increases.

1.3.2 Weight density or Specific weight (γ):

Weight density or Specific weight of a fluid is the weight per unit volume.

$$\therefore \quad \gamma = \frac{Weight}{Volume} = \frac{W}{V} \text{ or } \frac{dW}{dV}$$

Unit: N/m^3 or Nm^{-3}.

With increase in temperature volume increases and hence specific weight decreases.

With increases in pressure volume decreases and hence specific weight increases.

Note: Relationship between mass density and weight density:

We have $\gamma = \dfrac{Weight}{Volume}$

$$\gamma = \frac{mass\ x\ g}{Volume}$$

$$\gamma = \rho \text{ x } g$$

1.3.3 Specific gravity or Relative density (S):

It is the ratio of density of the fluid to the density of a standard fluid.

$$S = \frac{\rho_{fluid}}{\rho_{standard fluid}}$$

Unit: It is a dimensionless quantity and has no unit.

In case of liquids water at 4°C is considered as standard liquid. $\rho_{water} = 1000$ kg/m^3

1.3.4 Specific volume ($\forall$): It is the volume per unit mass of the fluid.

$$\therefore \quad \forall = \frac{Volume}{mass} = \frac{V}{M} \text{ or } \frac{dV}{dM}$$

Unit: m^3/kg

As the temperature increases volume increases and hence specific volume increases. As the pressure increases volume decreases and hence specific volume decreases.

Solved Problems:

Ex.1 Calculate specific weight, mass density, specific volume and specific gravity of a liquid having a volume of $4m^3$ and weighing 29.43 kN. Assume missing data suitably.

$\gamma = ?$

$\rho = ?$

$\forall = ?$

$S = ?$

$V = 4\,m^3$

$W = 29.43\ kN$

$= 29.43 \times 10^3\,N$

$$\gamma = \frac{W}{V}$$

$$= \frac{29.43 X 10^3}{4}$$

$$\gamma = 7357.58\ N/m^3$$

To find ρ - Method 1:

$$W = mg$$

$$29.43 \times 10^3 = m \times 9.81$$

$$m = 3000\,kg$$

$$\therefore \rho = \frac{m}{v} = \frac{3000}{4}$$

$$\rho = 750\,kg/m^3$$

Method 2:

$$\gamma = \rho\, g$$

$$7357.5 = \rho\ 9.81$$

$$\rho = 750\ kg/m^3$$

i) $\forall = \dfrac{V}{M}$

$$= \frac{4}{3000}$$

$$\forall = 1.33 \times 10^{-3}\,m^3/kg$$

$$\rho = \frac{M}{V}$$

$$\forall = \frac{V}{M}$$

$$\forall = \frac{1}{\rho} = \frac{1}{750}$$

$$\forall = 1.33X10^{-3}\,m^3/kg$$

$$S = \frac{\gamma}{\gamma_{Standard}} \qquad S = \frac{\rho}{\rho_{Standaard}}$$

$$= \frac{7357.5}{9810} \qquad \text{or} \qquad S = \frac{750}{1000}$$

$$S = 0.75 \qquad S = 0.75$$

Ex.2 Calculate specific weight, density, specific volume and specific gravity and if one liter of Petrol weighs 6.867N.

$$\gamma = \frac{W}{V}$$

$$V = 1\,\text{Litre}$$

$$= \frac{6.867}{10^{-3}}$$

$$V = 10^{-3}\,m^3$$

$$W = 6.867N$$

$$\gamma = 6867\,N/m^3$$

$$S = \frac{\gamma}{\gamma_{Standard}}$$

$$\rho = s\ g$$

$$= \frac{6867}{9810}$$

$$6867 = \rho \times 9.81$$

$$S = 0.7$$

$$\rho = 700\,kg/m^3$$

$$\forall = \frac{V}{M}$$

$$M = 6.867 \div 9.81$$

$$= \frac{10^{-3}}{0.7}$$

$$M = 0.7\ kg$$

$$\forall = 1.4x10^{-3}\,m^3/kg$$

Ex.3 Specific gravity of a liquid is 0.7 Find i) Mass density ii) specific weight. Also find the mass and weight of 10 Liters of liquid.

$$S = \frac{\gamma}{\gamma_{Standard}}$$

$$\gamma = \rho g$$

$$S = 0.7$$
$$V = ?$$
$$\rho = ?$$
$$M = ?$$
$$W = ?$$
$$V = 10 \text{ litre}$$
$$= 10x10^{-3} m^3$$

$$0.7 = \frac{\gamma}{9810}$$

$$6867 = \rho \text{ x } 9.81$$

$$\gamma = 6867 N/m^3$$

$$\rho = 700 \text{ kg}/m^3$$

$$S = \frac{\rho}{\rho_{Standard}}$$

$$0.7 = \frac{\rho}{1000}$$

$$\rho = 700 kg/m^3$$

$$\rho = \frac{M}{V}$$

$$700 = \frac{M}{10x10^{-3}}$$

$$M = 7kg$$

1.3.5 Viscosity: Viscosity is the property by virtue of which fluid offers resistance against the flow or shear deformation. In other words, it is the reluctance of the fluid to flow. Viscous force is that force of resistance offered by a layer of fluid for the motion of another layer over it.

In case of liquids, viscosity is due to cohesive force between the molecules of adjacent layers of liquid. In case of gases, molecular activity between adjacent layers is the cause of viscosity.

- **Newton's law of viscosity:**

Let us consider a liquid between the fixed plate and the movable plate at a distance 'Y' apart, 'A' is the contact area (Wetted area) of the movable plate, 'F' is the force required to move the plate with a velocity 'U' According to Newton's law shear stress is proportional to shear strain. (Fig.2)

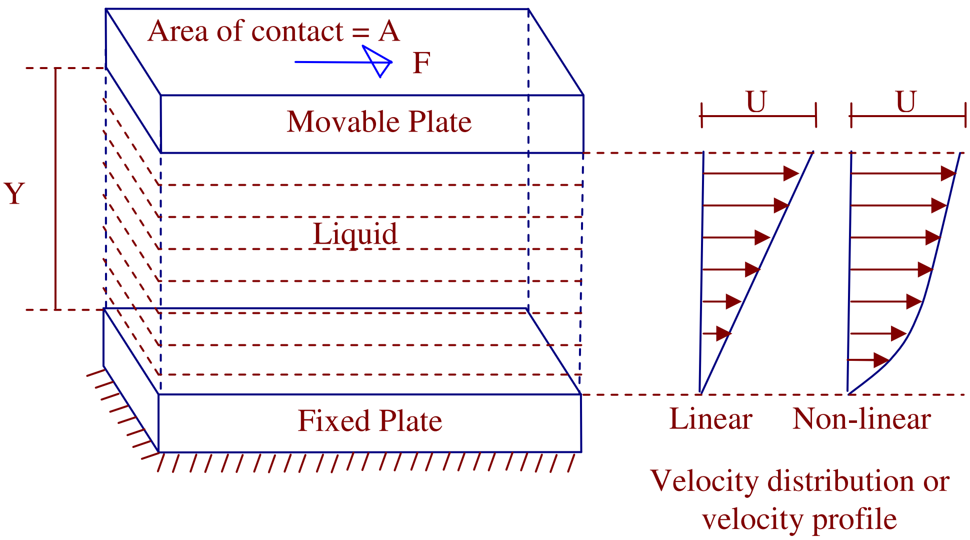


Fig.2 Definition diagram of Liquid viscosity

- $F \alpha A$
- $F \alpha \frac{1}{Y}$
- $F \alpha U$

$$\therefore F \alpha \frac{AU}{Y}$$

$$F = \mu . \frac{AU}{Y}$$

'μ' is the constant of proportionality called Dynamic Viscosity or Absolute Viscosity or Coefficient of Viscosity or Viscosity of the fluid.

$$\frac{F}{A} = \mu . \frac{U}{Y} \longrightarrow \therefore \tau = \mu \frac{U}{Y}$$

'τ' is the force required; Per Unit area called 'Shear Stress'. The above equation is called Newton's law of viscosity.

Velocity gradient or rate of shear strain:

It is the difference in velocity per unit distance between any two layers.

If the velocity profile is linear then velocity gradient is given by $\frac{U}{Y}$. If the velocity profile is non – linear then it is given by $\frac{du}{dy}$.

- Unit of force (F): N.
- Unit of distance between the twp plates (Y): m
- Unit of velocity (U): m/s
- Unit of velocity gradient : $\frac{U}{Y} = \frac{m/s}{m} = /s = s^{-1}$
- Unit of dynamic viscosity (τ): $\tau = \mu \cdot \frac{u}{y}$

$$\mu = \frac{\tau y}{U}$$

$$\Rightarrow \frac{N/m^2 \cdot m}{m/s}$$

$$\mu \Rightarrow \frac{N\text{-}sec}{m^2} \quad \text{or} \quad \mu \Rightarrow P_a\text{-}S$$

NOTE: In CGS system unit of dynamic viscosity is $\frac{dyne \cdot S}{Cm^2}$ and is called poise (P).

If the value of μ is given in poise, multiply it by 0.1 to get it in $\frac{NS}{m^2}$.

1 Centipoises = 10^{-2} Poise.

- **Effect of Pressure on Viscosity of fluids:**

  Pressure has very little or no effect on the viscosity of fluids.

- **Effect of Temperature on Viscosity of fluids:**

  1. *Effect of temperature on viscosity of liquids:* Viscosity of liquids is due to cohesive force between the molecules of adjacent layers. As the temperature increases cohesive force decreases and hence viscosity decreases.
  2. *Effect of temperature on viscosity of gases:* Viscosity of gases is due to molecular activity between adjacent layers. As the temperature increases molecular activity increases and hence viscosity increases.

- **Kinematics Viscosity:** It is the ratio of dynamic viscosity of the fluid to its mass density.

$$\therefore \text{Kinematic Viscosity} = \frac{\mu}{\rho}$$

Unit of KV:

$$KV \Rightarrow \frac{\mu}{\rho}$$

$$\Rightarrow \frac{NS/m^2}{kg/m^3}$$

$$= \frac{NS}{m^2} x \frac{m^3}{kg}$$

$$= \left(\frac{kg\,m}{s^2}\right) x \frac{s}{m^2} x \frac{m^3}{kg} = m^2/s$$

$$F = ma$$

$$N = Kg.m/s^2$$

$$\therefore \text{Kinematic Viscosity} = m^2/s$$

NOTE: Unit of kinematics Viscosity in CGS system is <u>cm^2/s</u> and is called <u>stoke (S)</u>

If the value of KV is given in stoke, multiply it by 10^{-4} to convert it into m^2/s.

The Fig. 3 illustrates how μ changes for different fluids.

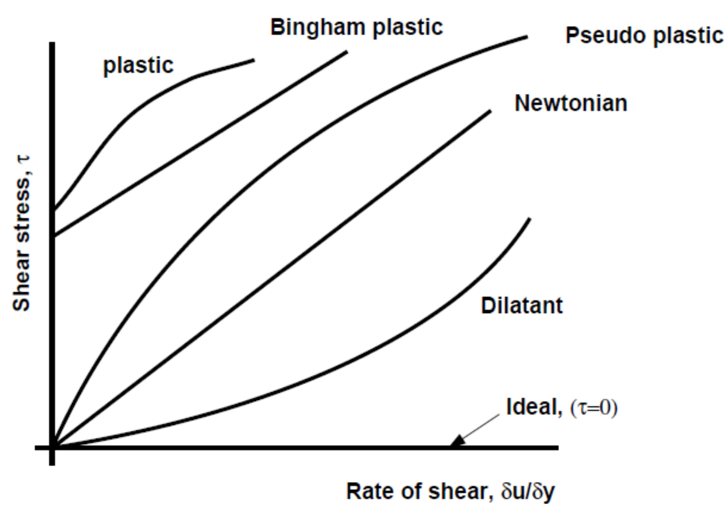
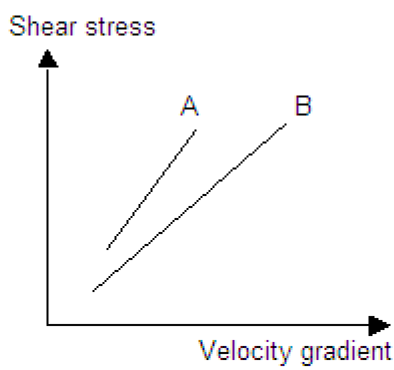


Fig.3 Variation of Viscosity based on Behaviour of Liquids

- Plastic: Shear stress must reach a certain minimum before flow commences.
- Bingham plastic: As with the plastic above a minimum shear stress must be achieved. With this classification n = 1. An example is sewage sludge.
- Pseudo-plastic: No minimum shear stress necessary and the viscosity decreases with rate of shear, e.g. colloidal substances like clay, milk and cement.
- Dilatant substances; Viscosity increases with rate of shear e.g. quicksand.
- Thixotropic substances: Viscosity decreases with length of time shear force is applied e.g. thixotropic jelly paints.
- Rheopectic substances: Viscosity increases with length of time shear force is applied
- Viscoelastic materials: Similar to Newtonian but if there is a sudden large change in shear they behave like plastic

The figure shows the relationship between shear stress and velocity gradient for two fluids, A and B. Comment on the Liquid 'A' and Liquid 'B' ?



Comment: (i) The dynamic viscosity of liquid A > the dynamic viscosity of liquid B
(ii) Both liquids follow Newton's Law of Vsiscoity

Solved Problems:

1. Viscosity of water is 0.01 poise. Find its kinematics viscosity if specific gravity is 0.998.

Kinematics viscosity = ?

S = 0.998

$$S = \frac{\rho}{\rho_{s\tan drad}}$$

$$0.998 = \frac{\rho}{1000}$$

$$\rho = 998 \text{ kg/m}^3$$

$\mu = 0.01P$

$= 0.01x0.1$

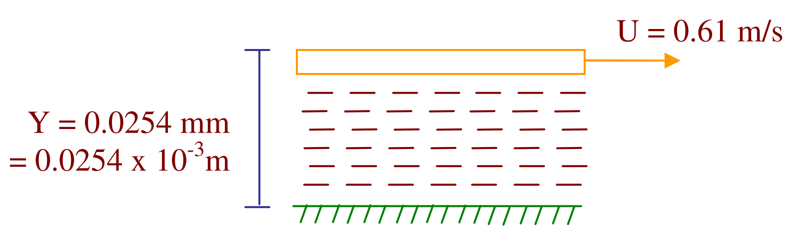
$$\mu = 0.001 \frac{NS}{m^2}$$

$$\therefore \text{Kinmetic Vis}\cos\text{ity} = \frac{\mu}{\rho}$$

$$= \frac{0.001}{998}$$

$$KV = 1 \text{ x } 10^{\ 6} \text{m}^2/\text{s}$$

2. A Plate at a distance 0.0254mm from a fixed plate moves at 0.61m/s and requires a force of 1.962N/m^2 area of plate. Determine dynamic viscosity of liquid between the plates.



$$\tau = 1.962 \text{ N/m}^2$$

$\mu = ?$

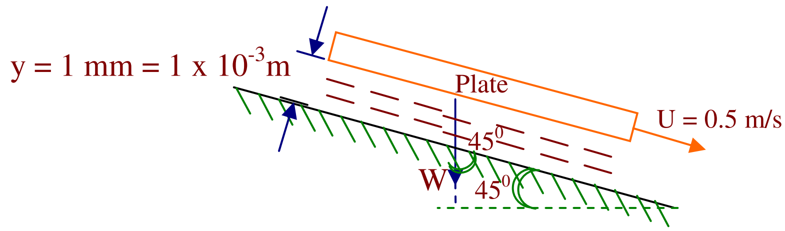
Assuming linear velocity distribution

$$\tau = \mu \frac{U}{Y}$$

$$1.962 = \mu \text{ x } \frac{0.61}{0.0254 \text{ x } 10^{-3}}$$

$$\mu = 8.17 \text{ x } 10^{-5} \frac{\text{NS}}{\text{m}^2}$$

3. A plate having an area of $1m^2$ is dragged down an inclined plane at 45^0 to horizontal with a velocity of 0.5m/s due to its own weight. Three is a cushion of liquid 1mm thick between the inclined plane and the plate. If viscosity of oil is 0.1 PaS find the weight of the plate.



$A = 1m^2$

$U = 0.5 m/s$

$Y = 1x10^{-3} m$

$\mu = 0.1 NS/m^2$

$W = ?$

$F = W \times \cos 45^0$

$= W \times 0.707$

$F = 0.707W$

$$\tau = \frac{F}{A}$$

$$\tau = \frac{0.707W}{1}$$

$$\tau = 0.707 W N/m^2$$

Assuming linear velocity distribution,

$$\tau = \mu . \frac{U}{Y}$$

$$0.707W = 0.1 \times \frac{0.5}{1 \times 10^{-3}}$$

$$W = 70.72 N$$

4. A flat plate is sliding at a constant velocity of 5 m/s on a large horizontal table. A thin layer of oil (of absolute viscosity = 0.40 N-s/m^2) separates the plate from the table. Calculate the thickness of the oil film (mm) to limit the shear stress in the oil layer to 1 kPa,

   Given : τ = 1 kPa = 1000 N/m2; U = 5m/s; μ = 0.4 N-s/m^2

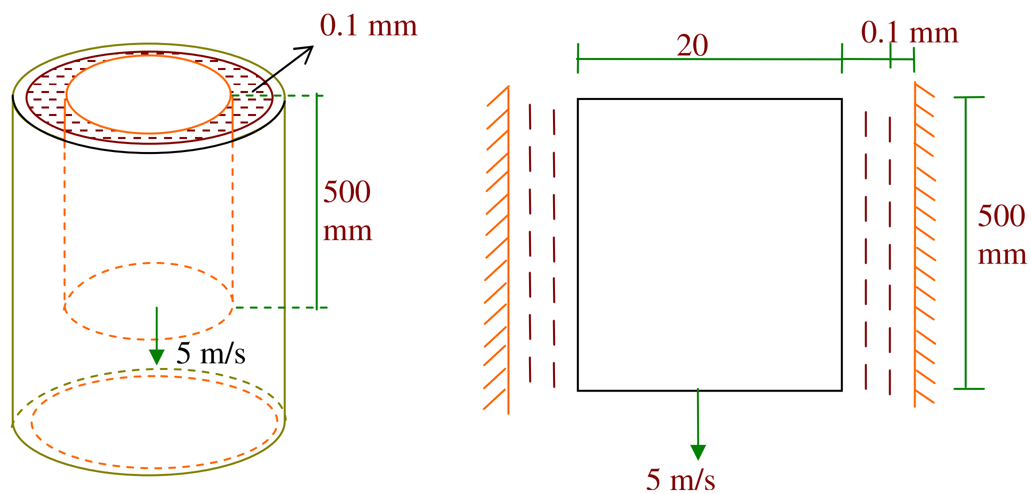
   Applying Newton's Viscosity law for the oil film -

$$\tau = \mu \frac{du}{dy} = \mu \frac{U}{y}$$

$$1000 = 0.4 \frac{5}{y}$$

$$y = 2 \times 10^{-3} = 2 \text{ mm}$$

5. A shaft of ϕ 20mm and mass 15kg slides vertically in a sleeve with a velocity of 5 m/s. The gap between the shaft and the sleeve is 0.1mm and is filled with oil. Calculate the viscosity of oil if the length of the shaft is 500mm.



D = 20mm = 20x10^{-3}m

M = 15 kg

W = 15x 9.81

<u>W = 147.15N</u>

y = 0.1mm

<u>y = 0.1 x 10^{-3}mm</u>

U = 5m/s

F = W

F = 147.15N

μ= ?

A = Π D L

A = Π x 20 x 10^{-3} x 0.5

A = 0.031 m^2

$$\tau = \mu . \frac{U}{Y}$$

$$4746.7 = \mu \, x \frac{5}{0.1 x 10^{-3}}$$

$$\mu = 0.095 \ \frac{NS}{m^2}$$

$$\tau = \frac{F}{A}$$

$$= \frac{147.15}{0.031}$$

$$\tau = 4746.7 N / m^2$$

6. If the equation of velocity profile over 2 plate is V= $2y^{2/3}$. in which 'V' is the velocity in m/s and 'y' is the distance in 'm' . Determine shear stress at (i) y = 0 (ii) y = 75mm. Take μ = 8.35P.
   a. at y = 0
   b. at y = 75mm

      = 75 x 10^{-3}m

      τ = 8.35 P

      = 8.35 x 0.1 $\frac{NS}{m^2}$

      $= 0.835 \ \frac{NS}{m^2}$

$$V = 2y^{2/3}$$

$$\frac{dv}{dy} = 2x\,\frac{2}{3}\,y^{2/3-1}$$

$$= \frac{4}{3}\,y^{-1/3}$$

$$at,\ y = 0, \frac{dv}{dy} = 3\frac{4}{\sqrt[3]{0}} = \infty$$

$$at,\ y = 75x10^{-3}\,m, \frac{dv}{dy} = 3\frac{4}{\sqrt[3]{75x10^{-3}}}$$

$$\frac{dv}{dy} = 3.16\,/\,s$$

$$\tau = \mu.\frac{dv}{dy}$$

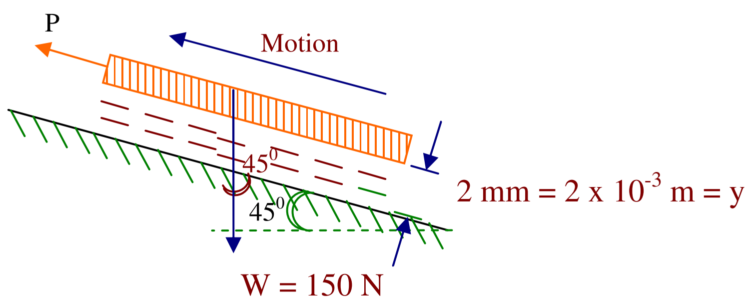
$$at,\ y = 0,\ \ \tau = 0.835x\infty$$

$$\tau = \infty$$

$$at,\ y = 75\,x10^{-3}\,m, \tau = 0.835\ x\ 3.16$$

$$\tau = 2.64\ N\,/\,m^2$$

7. A circular disc of 0.3m dia and weight 50 N is kept on an inclined surface with a slope of 45^0. The space between the disc and the surface is 2 mm and is filled with oil of dynamics viscosity $\frac{1NS}{m^2}$. What force will be required to pull the disk up the inclined plane with a velocity of 0.5m/s.



D = 0.3m

$$A = \frac{\prod x \; 0.3m^2}{4}$$

$A = 0.07m^2$

W = 50N

$$\mu = 1 \; \frac{NS}{m^2}$$

$$\frac{y = 2x10^{-3} m}{U = 0.5m/s}$$

$$F = P - 50\cos 45$$

$$F = (P - 35{,}35)$$

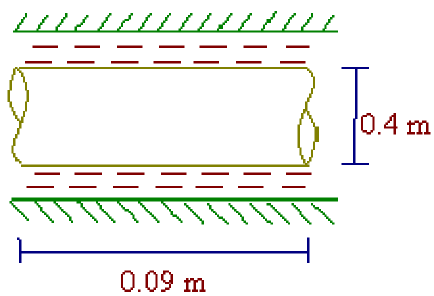
$$\upsilon = \frac{(P - 35.35)}{0.07} N/m^2$$

$$\tau = \mu . \frac{U}{Y}$$

$$\left(\frac{P - 35{,}35}{0.07} \right) = 1x \; \frac{0.5}{2x10^{-3}}$$

$$P = 52.85N$$

8. Dynamic viscosity of oil used for lubrication between a shaft and a sleeve is 6 P. The shaft is of diameter 0.4 m and rotates at 190 rpm. Calculate the power lost in the bearing for a sleeve length of 0.09 m .Thickness of oil is 1.5 mm.



$\mu = 6 = 0.6\frac{NS}{m^2}$

N = 190 rpm

Power lost = ?

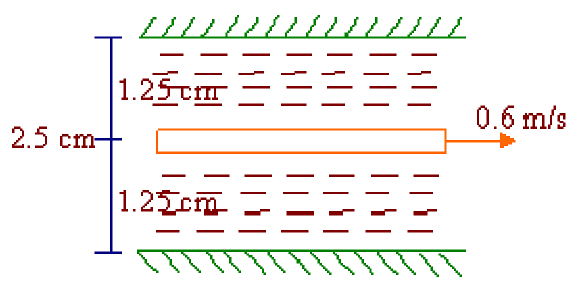
A = Π D L

= Π x 0.4 x0.09   A = 0.11m^2

Y = 1.5x10^{-3} m

9. Two large surfaces are 2.5 cm apart. This space is filled with glycerin of absolute viscosity 0.82 NS/m^2. Find what force is required to drag a plate of area 0.5m^2 between the two surfaces at a speed of 0.6m/s. (i) When the plate is equidistant from the surfaces, (ii) when the plate is at 1cm from one of the surfaces.

Case (i) When the plate is equidistant from the surfaces,



$$U = \frac{\Pi DN}{60}$$

$$= \frac{\Pi \times 0.4 \times 190}{60}$$

$$U = 3.979 \ m/s$$

$$\tau = \mu . \frac{U}{Y}$$

$$= 0.6 \times \frac{3.979}{1.5 \times 10^{-3}}$$

$$\tau = 1.592 \times 10^{3} N/m^{2}$$

$$\frac{F}{A} = 1.59 \times 10^{3}$$

$$F = 1.591 \times 10^{3} \times 0.11$$

$$F = 175.01 \ N$$

$$T = F x R$$

$$= 175.01 x 0.2$$

$$T = 35 Nm$$

$$P = \frac{2\Pi NT}{60,000}$$

$$P = 0.6964 KW$$

$$P = 696.4 W$$

Let F_1 be the force required to overcome viscosity resistance of liquid above the plate and F_2 be the force required to overcome viscous resistance of liquid below the plate. In this case $F_1 = F_2$. Since the liquid is same on either side or the plate is equidistant from the surfaces.

$$\tau_1 = \mu_1 \frac{U}{Y}$$

$$\tau_1 = 0.82x \frac{0.6}{0.0125}$$

$$\tau_1 = 39.36 N/m^2$$

$$\frac{F_1}{A} = 39.36$$

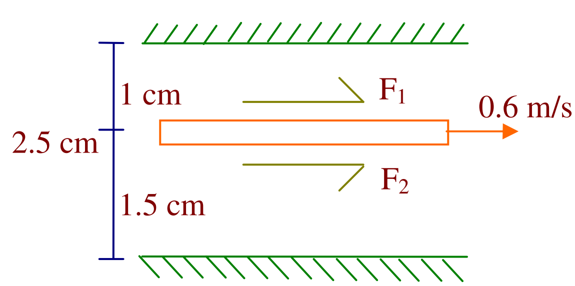
$$F_1 = 19.68N$$

∴ Tatal force required to drag the plate $=F_1 + F_2 = 19.68+19.68$

$F= 39.36N$

Case (ii) when the plate is at 1cm from one of the surfaces.

Here $F_1 \neq F_2$



$$\frac{F_1}{A} = 49.2$$

$$F_1 = 49.2 \times 0.5$$

$$F_1 = 24.6\ N$$

$$\frac{F_2}{A} = 32.8$$

$$F_2 = 32.8 \times 0.5$$

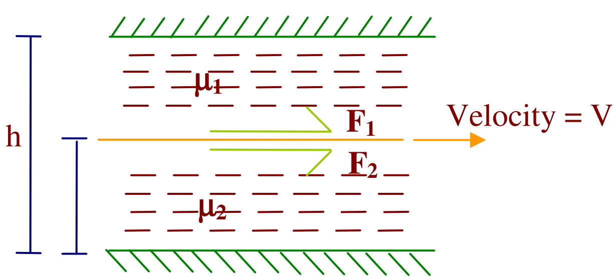
$$F_2 = 16.4\ N$$

Total Force $F = F_1 + F_2 = 24.6 + 16.4$

$F = 41N$

10. Through a very narrow gap of ht a thin plate of large extent is pulled at a velocity `V'. On one side of the plate is oil of viscosity μ_1 and on the other side there is oil of viscosity μ_2. Determine the position of the plate for the following conditions.
    i. Shear stress on the two sides of the plate is equal.
    ii. The pull required, to drag the plate is minimum.

Condition 1: Shear stress on the two sides of the plate is equal $F_1 = F_2$



$y = ?$ for $F_1 = F_2$

$$\tau = \mu . \frac{U}{Y}$$

$$\frac{F}{A} = \mu . \frac{U}{Y}$$

$$F = A\mu . \frac{U}{Y}$$

$$F_1 = \frac{A \mu_1 V}{(h - y)}$$

$$F_2 = \frac{A \mu_2 V}{y}$$

$$F_1 = F_2$$

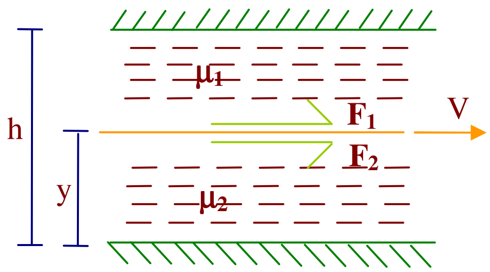
$$\frac{A\mu_1 V}{h-y} = \frac{A\mu_2 V}{y}$$

$$\mu_1 y = \mu_2 (h-y)$$

$$\mu_1 y + \mu_2 y = \mu_2 h$$

$$y = \frac{\mu_2 h}{\mu_1 + \mu_2} \text{ or } y = \frac{h}{\frac{\mu_1}{\mu_2} + 1}$$

Condition 2: The pull required, to drag the plate is minimum $\left(\text{i.e.}\left[\frac{dF}{dy}\right]_{min\,imum}\right)$



$y = ?$ if, $F_1 + F_2$ is to be minimum

$$F_1 = \frac{A\mu_1 V}{h \quad y}$$

$$F_2 = \frac{A\mu_2 V}{y}$$

$\therefore$ Total drag forced required

$$F = F_1 + F_2$$

$$F = \frac{A\mu_1 V}{h \quad y} + \frac{A\mu_2 V}{y}$$

For F to be min . $\frac{dF}{dy} = 0$

$$\frac{dF}{dy} = 0 = +A\mu_1 V \equiv (h \quad y)^2 \quad A\mu_2 V y^2$$

$$= \frac{V\mu_1 A}{(h \quad y)^2} \quad \frac{V\mu_2 A}{y^2}$$

$$\frac{(h \quad y)^2}{y^2} = \frac{\mu_1}{\mu_2}$$

$$\frac{h \quad y}{y} = \sqrt{\frac{\mu_1}{\mu_2}}$$

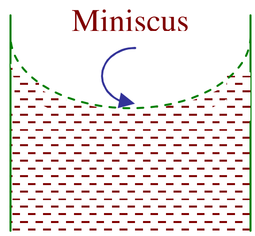
$$(h \quad y) = y\sqrt{\frac{\mu_1}{\mu_2}}$$

$$h = y\sqrt{\frac{\mu_1}{\mu_2}} + y$$

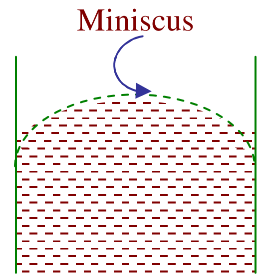
$$h = y \quad 1 + \sqrt{\frac{\mu_1}{\mu_2}}$$

$$\therefore \; y = \frac{h}{1 + \sqrt{\frac{\mu_1}{\mu_2}}}$$

1..3.6 Capillarity :

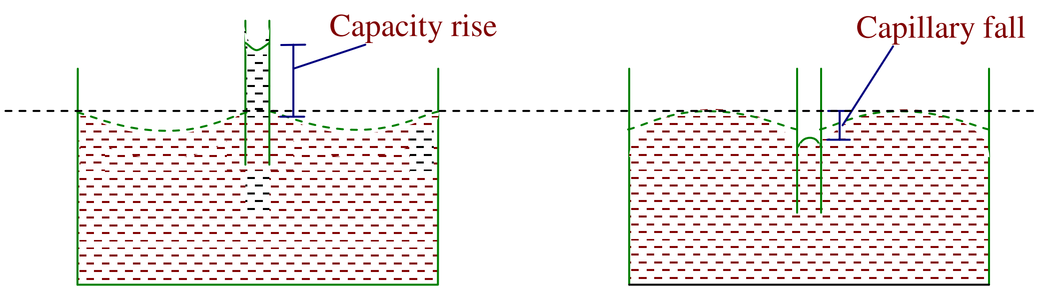


Cohesion < Adhesion
Eg: Water



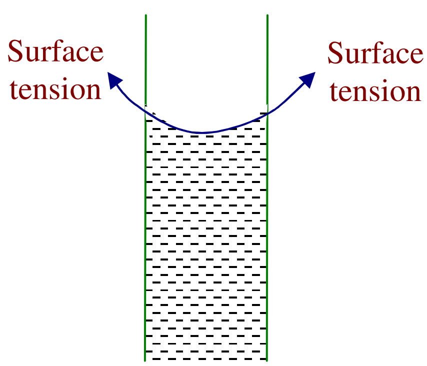
Cohesion > Adhesion
Eg: Mercury

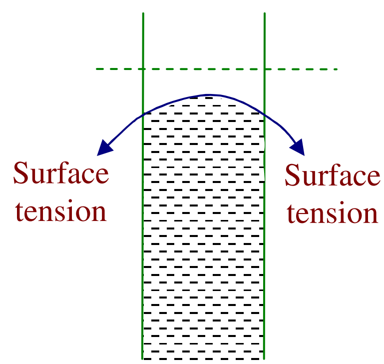
Any liquid between contact surfaces attains curved shaped surface as shown in figure. The curved surface of the liquid is called Meniscus. If adhesion is more than cohesion then the meniscus will be concave. If cohesion is greater than adhesion meniscus will be convex.



Cohesion < Adhesion
Eg: Water

Cohesion > Adhesion
Eg: Mercury

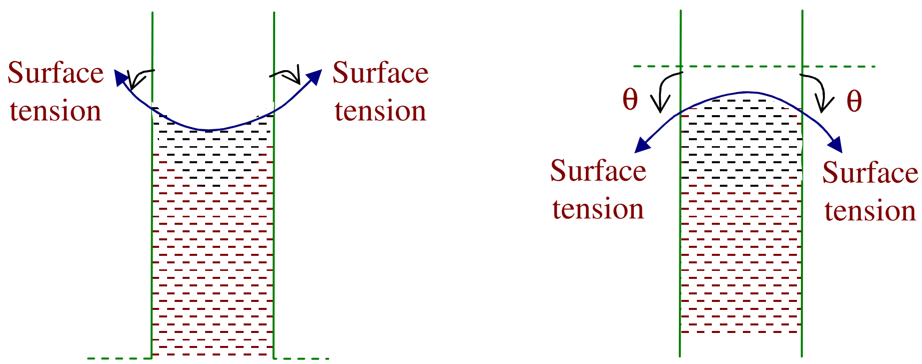




Capillarity is the phenomena by which liquids will rise or fall in a tube of small diameter dipped in them. Capillarity is due to cohesion adhesion and surface tension of liquids. If adhesion is more than cohesion then there will be capillary rise. If cohesion is greater than adhesion then will be capillary fall or depression. The surface tensile force supports capillary rise or depression.

Note:

Angle of contact:



θ → Angle of contact
→ Acute

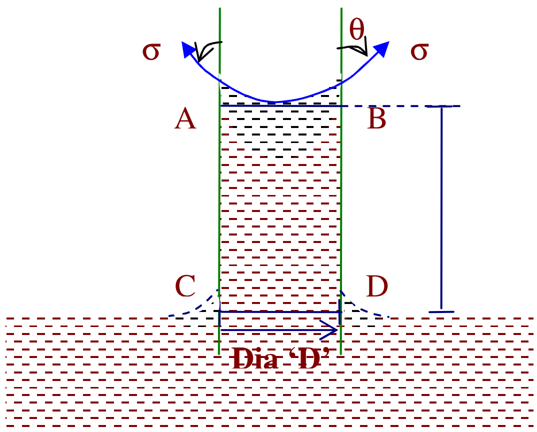
θ → Angle of contact
→ Obtuse

The angle between surface tensile force and the vertical is called angle of contact. If adhesion is more than cohesion then angle of contact is obtuse.

- **To derive an expression for the capillary rise of a liquid in small tube dipped in it:**

Let us consider a small tube of diameter 'D' dipped in a liquid of specific weight γ. 'h' is the capillary rise. For the equilibrium,

Vertical force due to surface tension = Weight of column of liquid ABCD



$$[\sigma(\Pi D)]\cos\theta = \gamma \text{ x volume}$$

$$[\sigma(\Pi D)]\cos\theta = \gamma \text{ x } \frac{\Pi D^2}{4} \text{ x } h$$

$$h = \frac{4\sigma\cos\theta}{\gamma D}$$

It can be observed that the capillary rise is inversely proportional to the diameter of the tube.

Note:

The same equation can be used to calculate capillary depression. In such cases 'θ' will be obtuse 'h' works out to be –ve.

Problems:

1. Capillary tube having an inside diameter 5mm is dipped in water at 20^0. Determine the heat of water which will rise in tube. Take σ =0.0736N/m at 20^0 C.

$$h = \frac{4\sigma\cos\theta}{\gamma D}$$

$$= \frac{4 \text{ x } 0.0736 \text{ x } \cos\theta}{9810 \text{ x } 5 \text{ x } 10^{-3}}$$

$$\theta = 0^0 \text{ (assumed)}$$

$$\gamma = 9810 N/m^3$$

$$h = 6 \text{ x } 10^{-3} m$$

2. Calculate capillary rise in a glass tube when immersed in Hg at 20^0c. Assume σ for Hg at 20^0c as 0.51N/m. The diameter of the tube is 5mm. $\theta = 130^0$c.

$$S = \frac{\gamma}{\gamma_{Standard}}$$

$$h = \frac{4\sigma\cos\theta}{\gamma D}$$

$$13.6 = \frac{\gamma}{9810}$$

$$h = -1.965 \text{ x} 10^{-3} m$$

$$\gamma = 133.416 \text{ x } 10^3 N/m^3$$

-ve sign indicates capillary depression.

3. Determine the minimum size of the glass tubing that can be used to measure water level if capillary rise is not to exceed 2.5mm. Take σ = 0.0736 N/m.

$$h = \frac{4\sigma \cos\theta}{\gamma D}$$

$$D = \frac{4 \times 0.0736 \times \cos 0}{9810 \times 2.5 \times 10^{-3}}$$

$D = ?$

$h = 2.5 \times 10^{-3}\,m$

$$D = 0.012\,m$$

$\sigma = 0.0736\ N/m$

$$D = 12\,mm$$

4. A glass tube 0.25mm in diameter contains Hg column with air above it. If σ = 0.51N/m, what will be the capillary depression? Take θ = - 40^0 or 140^0.

$$h = \frac{4\sigma \cos\theta}{\gamma D}$$

$D = 0.25 \times 10^{-3}\,m$

$\sigma = 0.51 N/m$

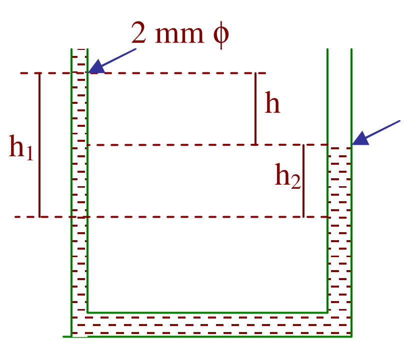
$$= \frac{4 \times 0.51 \times \cos 140}{133.146 \times 10^{-3} \times 0.25 \times 10^{-3}}$$

$\theta = 140$

$$h = -46.851 \times 10^{-3}\,m$$

$\gamma = 133.416 \times 10^3\ N/m^2$

5. If a tube is made so that one limb is 20mm in ϕ and the other 2mm in ϕ and water is poured in the tube, what is the difference in the level of surface of liquid in the two limbs. σ = 0.073 N/m for water.



$$h_1 = h = \frac{4\sigma\cos\theta}{\gamma D}$$

$$= \frac{4 \text{ x } 0.073 \text{ x } \cos o}{9810 \text{ x } (20 \text{x} 10^{-3})}$$

$$= 0.01488 m$$

$$h_2 = \frac{4 \text{ x } 0.073 \text{x} \cos o}{9810 \text{ x } (20 \text{x} 10^{-3})}$$

$$= 1.488 \text{ x } 10^{-3} m$$

$$h = h_1 - h_2$$

$$= 0.01339 m$$

$$h = 13.39 mm$$

6. A clean glass tube is to be selected in the design of a manometer to measure the pressure of kerosene. Specific gravity of kerosene = 0.82 and surface tension of kerosene = 0.025 N/m. If the capillary rise is to be limited to 1 mm, the smallest diameter (cm) of the glass tube should be most nearly

Soln. Given For kerosene σ = 0.025 N/m ; Sp.Gr. = 0.82; h_{max} = 1mm

Assuming contact angle θ = 0°, $\gamma_{kerosene}$ =0 .82 x 9810 = 8044.2 N/m^3

Let 'd' be the smallest diameter of the glass tube in Cm

Then using formula for capillary rise in (h)

$$h = \frac{4\ \sigma\ \text{Cos}\theta}{\gamma_{kerosene}(\frac{d_{cm}}{100})} = \frac{4\text{x}0.025\ \text{Cos}0^\circ}{8044.2\text{x}(\frac{d_{cm}}{100})} = \frac{1}{1000}$$

$$d_{cm} = 1.24 \text{ Cm}$$