FLUID DYNAMICS

Fluid dynamics is that branch of fluid mechanics wherein we study the analysis of the fluid motion along with the forces generating them.

The fluid motion is analysed by the ***Newton's second law of motion,*** which states that t***he force applied on a body along any direction is given by the rate of change of momentum along the same direction***.

$$F_x = \frac{(m\mathrm{v}_2 - m\mathrm{v}_1)}{t} = ma \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (01)$$

The forces acting on the fluid can be classified as under:

1. ***$F_g \Rightarrow$ Gravity forces***
2. ***$F_P \Rightarrow$ Pressure forces***
3. ***$F_v \Rightarrow$ Viscous forces***
4. ***$F_t \Rightarrow$ Turbulent forces***
5. ***$_{Fe} \Rightarrow$ Elastic forces***
6. ***$F_c \Rightarrow$ Compressibility forces***

The net force acting along ***x*** direction is given by

$$F_x = (F_g)_x + (F_P)_x + (F_v)_x + (F_t)_x + (F_e)_x + (F_c)_x \qquad \ldots\ldots\ldots\ldots\ldots (02)$$

Equations of motion

Reynolds equation of motion: In the equation of motion (Eq. 01), the force due to compressibility is neglected and only forces due to gravity, pressure, viscosity and turbulence are considered, the resulting equation is termed as ***Reynolds equation of motion.***

Navier-Stokes equation of motion: In the equation of motion (Eq. 01), the forces due to compressibility and turbulence are neglected and only forces due to gravity, pressure and viscosity are considered, the resulting equation is termed as ***Navier-Stokes equation of motion.***

Euler's equation of motion: In the equation of motion (Eq. 01), if the fluid is ideal, then the forces due to compressibility, turbulence and viscosity are neglected and only forces due to gravity and pressure are considered, the resulting equation is termed as ***Euler's equation of motion.***

i.e. $F_x = ma_x = F_g + F_P$

Euler's Equation of motion:

Assumptions:

1. ***Only Gravitational and Pressure forces are considered***
2. ***Fluid motion along a stream line is considered***

Consider a stream line along direction ***x*** as shown in Fig. Consider a cylindrical fluid element of cross-sectional area ***dA*** and length ***ds*** along the stream line direction.

The forces acting on the fluid element are:

1. The pressure force ***pdA*** along the flow direction ***s***
2. The pressure force $[p+(\partial p/\partial s)ds]dA$ against the flow direction ***s***
3. Weight of the fluid element = $\rho\, g\, dA\, ds$ acting vertically downwards at an angle θ with the vertical.

The resultant force acting along the flow direction is given by

$$F_s = p\,dA - \left[p + \frac{\partial p}{\partial s}ds\right]dA - \rho\, g\, dA\, ds \cos\theta$$

$$F_s = -\left[\frac{\partial p}{\partial s}ds\right]dA - \rho\, g\, dA\, ds \cos\theta \qquad \ldots\ldots\ldots (01)$$

From Newton's second law of motion, the net force along s direction is

$$Fs = (\rho\, dAds)\, a_s \qquad \ldots\ldots\ldots (02)$$

Where a_s is the acceleration along the flow direction.

But $a_s = \frac{d\mathrm{v}}{dt}$ (**v** is a function of space and time)

From chain rule, we can write

$$a_s = \frac{d\,\text{v}}{ds}\frac{d\,s}{dt} + \frac{d\,\text{v}}{dt}$$

As $\text{v} = \frac{d\,s}{dt}$, we can simplify the above equation as

$$a_s = \text{v}\frac{d\,\text{v}}{ds} + \frac{d\,\text{v}}{dt}$$

For steady flow, $\frac{d\,\text{v}}{dt} = 0$

Hence $a_s = \text{v}\frac{d\,\text{v}}{ds}$ …(03)

From Eqs. 1,2 and 3, we get

$$-\left[\frac{\partial p}{\partial s}ds\right]dA - \rho\, g\, dA\, ds \cos\theta = \rho\, dA ds\; \text{v}\frac{\partial \text{v}}{\partial s} \qquad \div\, \rho\, dA\, ds$$

Simplifying, we get $\frac{1}{\rho}\frac{\partial p}{\partial s} + g\,\cos\theta + \text{v}\frac{\partial \text{v}}{\partial s} = 0$ But $\cos\theta = \frac{dz}{d\,s}$

Hence $\frac{1}{\rho}\frac{\partial p}{\partial s} + g\,\frac{dz}{d\,s} + \text{v}\frac{\partial \text{v}}{\partial s} = 0$

Since the variation is only along the stream line and is unidirectional, the partial derivatives can be written as normal derivatives and

$$\frac{dp}{\rho} + g\;dz + \text{v}\,d\,\text{v} = 0 \qquad \textit{(Euler's Equation)}$$

Bernoulli's Equation from Euler's Equation of Motion

Assumptions and Limitations

1. ***The fluid is ideal. i.e. the viscosity is zero***
2. ***The flow is steady***
3. ***The flow is incompressible***
4. ***The flow is irrotational or the flow is along a stream line***

From Euler's equation of motion, we have

$$\frac{dp}{\rho} + g\;dz + \text{v}\,d\,\text{v} = 0 \qquad \ldots(01)$$

Integrating the above expression

$$\int\frac{dp}{\rho} + \int g\;dz + \int \text{v}\,d\,\text{v} = \text{Constant}$$

As the flow is incompressible, ρ is constant and we get

$$\frac{p}{\rho} + g\;z + \frac{\text{v}^2}{2} = \text{Constant} \qquad \div\, g$$

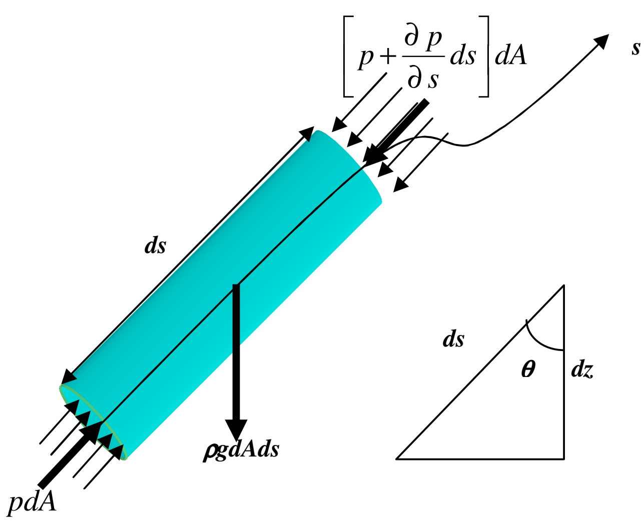
$$\frac{p}{\rho\,g} + \frac{\text{v}^2}{2\,g} + z = \text{Constant} \qquad \ldots(02)$$

The above equation is known as Bernoulli's energy equation in which

$\frac{p}{\rho\,g} \Rightarrow$ ***Pressure energy per unit weight or Pressure head***

$\frac{\text{v}^2}{2\,g} \Rightarrow$ ***Kinetic energy per unit weight or kinetic/velocity head***

$z \Rightarrow$ ***Potential energy per unit weight or potential/datum head***



Statement of Bernoulli's energy equation or theorem

In a steady, ideal flow of an incompressible fluid , the total energy at any section of a flowing fluid is always a constant. The total energy includes pressure energy, kinetic or velocity energy and potential or datum energy.

It can also be stated as in a steady, ideal flow of an incompressible fluid, the total head at any section of a flowing fluid is always a constant wherein the total head includes pressure head, velocity or kinetic head and datum or potential head.

Bernoulli's Equation for real fluid or Modified Bernoulli's Equation

The Bernoulli's equation has been derived for ideal and non-viscous fluid and hence it is frictionless. But, in case of real fluid, the viscosity will be very much present and hence, there will be energy or head loss between any two section along the flowing fluid. Hence, the Bernoulli's equation can be modified as

The total energy at any section of a flowing fluid is equal to the total energy at the previous section minus the energy loss between the two sections.

In other words, *The difference of total head between any two sections of a flowing fluid along the flow direction is given by the head loss between the two sections*. Mathematically, it can be stated as under:

$$\frac{p_A}{\gamma}+\frac{V_A^2}{2g}+Z_A=\frac{p_B}{\gamma}+\frac{V_B^2}{2g}+Z_B+h_L$$

Where h_L is the head loss or the energy loss per unit weight between the two sections of a flowing fluid.

Problems

Jan/Feb 2003

1. The following are the data given of a change in diameter, effected in laying a water supply pipe line. The change in diameter is gradual from 200 mm at A to 500 mm at B. Pressure at A and B are ***78.5 kN/m²*** and ***58.9 kn/m²*** respectively with the end B being ***3 m*** higher than A. If the glow in the pipe line is 200 lps, find:
   a) direction of flow and
   b) the head lost in friction between A and B. (10)

Solution:

$Q = 0.2$ m3/s; $D_A = 0.2$ m; $D_B = 0.5$ m; $g = 10$ m/s² (assumed)

$p_A = 78.5$ kN/m2; $p_B = 58.9$ kN/m2; $Z_B - Z_A = Z = 3$ m; $h_f = ?$

$$A_A=\frac{\pi}{4}D_A^2=0.0314\,\text{m}^2 \quad \text{and} \quad A_B=\frac{\pi}{4}D_B^2=0.1965\,\text{m}^2$$

From discharge continuity equation, we get

$$\text{v}_A=\frac{Q}{A_A}=6.3662\,\text{m/s} \quad \text{and} \quad \text{v}_B=\frac{Q}{A_B}=1.0186\,\text{m/s}$$

From modified Bernoulli's equation applied between A and B, we have

$$\frac{p_A}{\gamma}+\frac{V_A^2}{2g}+Z_A=\frac{p_B}{\gamma}+\frac{V_B^2}{2g}+Z_B+h_L$$

$$\frac{78.5\times10^3}{10\times10^3}+\frac{6.3662^2}{2\times10}=\frac{58.9\times10^3}{10\times10^3}+\frac{1.0186^2}{2\times10}+3+h_L$$

$$h_L = 0.9345\text{ m}$$

The flow is always from higher pressure to lower pressure and hence from ***A*** to ***B.***

Jan/Feb 2004

2. Water flows up a conical pipe ***450 mm*** diameter at the lower end and 2***50 mm*** diameter at ***2.3 m*** above the lower end. If the pressure and velocity at the lower end are ***63 kN/m²*** (gauge) and ***4.1 m/s***, assuming a head loss in the pipe to be ***10%*** of the pressure head at the lower end, calculate the discharge through the pipe. Also calculate the pressure and velocity at the upper end (8)

Solution

$D_A = 0.45$ m; $D_B = 0.25$ m; $Z_B - Z_A = Z = 2.3$ m; $V_A = 4.1$ m/s;

$p_A = 63$ kN/m² (gauge); $h_L = 10\%(p_A/\gamma)$; $Q = ?$; $p_B = ?$; $V_B = ?$

$\rho = 1000$ kg/m³, $g = 10$ m/s² (assumed)

From modified Bernoulli's equation applied between A and B, we have

$$\frac{p_A}{\gamma}+\frac{V_A^2}{2g}+Z_A=\frac{p_B}{\gamma}+\frac{V_B^2}{2g}+Z_B+h_L$$

$$A_A=\frac{\pi}{4}D_A^2=0.1590\text{m}^2 \text{ and } \qquad A_B=\frac{\pi}{4}D_B^2=0.0491\text{m}^2$$

From discharge continuity equation, we have $\boldsymbol{Q = A_A V_A = A_B V_B}$

$$\text{v}_B=\frac{A_B}{A_B}\text{v}_A=13.284\,\text{m/s}$$

$$h_L=0.1\times\frac{63\times10^3}{10^3\times10}=0.63\,\text{m}$$

Substituting in the modified Bernoulli's equation, we get

$$\frac{63\times10^3}{10\times10^3}+\frac{4.1^2}{2\times10}=\frac{p_B}{10\times10^3}+\frac{13.284^2}{2\times10}+2.3+0.63$$

Simplifying, we get p_B = ***-102.83*** kN/m^2 ***(gauge)***

3. A pipe 400 mm diameter carries water at a velocity of 2.5 m/s. The pressure head at pints A and B are given as 30 m and 23 m respectively, while the datum head at A and B are 28 m and 30 m respectively. Find the loss of head between A and B. (Jan/Feb 2005)

Solution:

D = *0.4* m; V = *2.5* m/s; $\frac{p_A}{\rho g}=h_A$ = *30* m; $\frac{p_B}{\rho g}=h_B$ = *23* m; Z_A = *28* m;

Z_B = *30* m; h_L = *?*

As the pipe is of uniform diameter $V_A = V_B$ = *2.5* m/s

Applying modified Bernoulli's equation between A and B

$$\frac{p_A}{\gamma}+\frac{V_A^2}{2g}+Z_A=\frac{p_B}{\gamma}+\frac{V_B^2}{2g}+Z_B+h_L$$

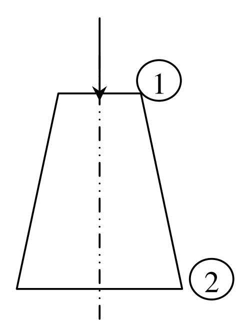
$$30+\frac{V^2}{2g}+28=23+\frac{V^2}{2g}+30+h_L$$

h_L = *5 m*

4. A conical tube of length *2* m is fixed vertically with its smaller end upwards. The velocity of flow at the smaller end is *5* m/s while at the lower end it is *2* m/s. The pressure head at the smaller end is *2.5* m/s of liquid. The loss of head in the tube is $0.35\frac{(\text{v}_1-\text{v}_2)^2}{2g}$, where v_1 is the velocity at the smaller end and v_2 is the velocity at the lower end respectively. Determine the pressure head at the lower end. Flow takes place in the downward direction.

Solution:

L = 2 m***,*** $\mathbf{v_1}$ ***= 5*** m/s***,*** $\mathbf{v_2}$ ***= 2*** m/s, ***g = 10*** m/s^2 $\frac{p_1}{\rho g}$ = ***2.5*** m***,*** $\frac{p_2}{\rho g}=?$



Let the smaller end be represented as 1 and lower end as 2 as shown in Fig.

$$h_L=\frac{0.35(\text{v}_1-\text{v}_2)^2}{2g}=\frac{0.35(5-2)^2}{2g}=0.16\,m$$

Applying modified Bernoulli's equation between 1 and 2

$$\frac{p_1}{\gamma}+\frac{V_1^2}{2g}+Z_1=\frac{p_2}{\gamma}+\frac{V_2^2}{2g}+Z_2+h_L$$

$$2.5+\frac{5^2}{2g}+2=\frac{p_2}{\gamma}+\frac{2^2}{2g}+0+0.16$$

$\frac{p_2}{\rho g}$ = ***5.39 m***

Applications of Bernoulli's Energy Equation.

Bernoulli's energy equation can be applied in all problems of fluid flow, wherein energy considerations are involved. Some of its applications are:

1. Venturimeter
2. Pitot tube.

Momentum equation

It is based on the law of conservation of momentum or on momentum principle, which states that the net force acting on a fluid mass is equal to the rate of change of momentum of flow in that direction. If the force acting on a mass of fluid m is F_x along x direction, the net force along the direction is given by Newton's second law of motion as $F_x = m\, a_x$

Where a_x is the acceleration produced due to the force F_x along the same direction.

But $a_x = \frac{du}{dt}$

Hence $F_x = ma_x = m\frac{du}{dt} = \frac{d(mu)}{dt}$ (as m is constant for incompressible flow)

The above equation is called momentum principle. The same equation can also be written as $F_x dt = d(mu)$ which is known as ***impulse momentum principle*** and can be stated as "The impulse of a force acting on a fluid of mass m in a short interval of time dt along any direction is given by the rate of change of momentum $d(mu)$ along the same direction.

Force exerted by a Flowing fluid on a pipe Bend

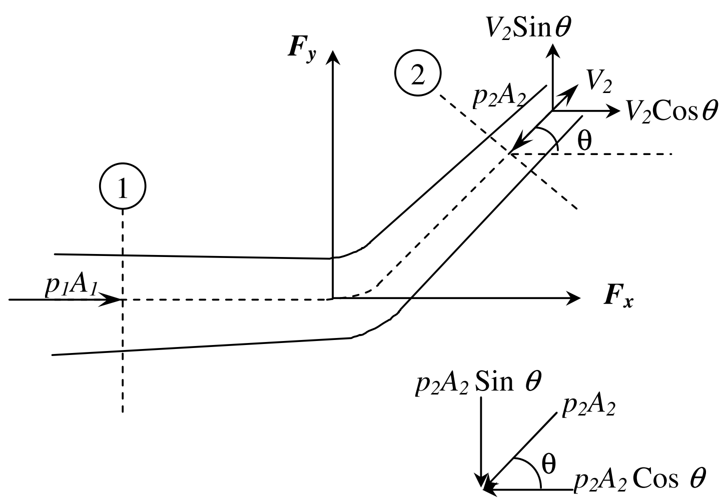
Consider a flow occurring in a pipe bend which is changing its cross sectional area along the bend as shown in the Fig. Let θ be the angle of bend and F_x and F_y be the force exerted by the fluid on the bend along the x and y directions respectively. The force exerted by the bend on the mass of fluid is $-F_x$ and $-F_y$. The other forces acting on the mass of fluid are hydrostatic pressure forces at the two sections 1 and 2 p_1A_1 along the flow direction and p_2A_2 against the flow direction respectively. From the momentum equation, the net force acting on the fluid mass along x direction is given by the rate of change of momentum in x direction.

i.e. $p_1A_1 - p_2A_2 \cos\theta - F_x$ =(Mass per second) (change in velocity)

$= \rho Q$ (Final velocity – initial velocity) along x

$= \rho Q (V_2 \cos\theta - V_1)$

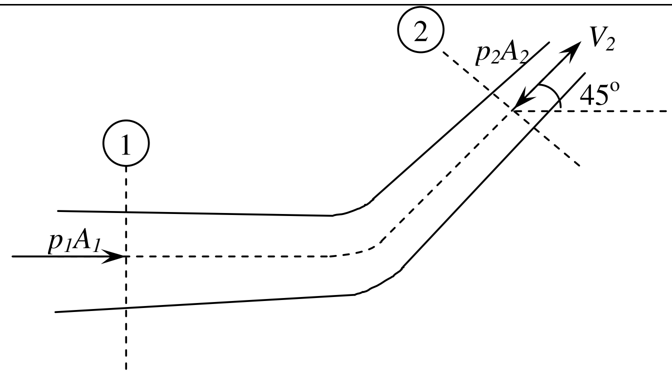
$$F_x = \rho Q (V_1 - V_2 \cos\theta) + p_1A_1 - p_2A_2 \cos\theta \quad \ldots(01)$$



Similarly the momentum equation in y direction gives

$$0 - p_2A_2 \sin\theta - F_y = \rho Q (V_2 \sin\theta - 0)$$

$$F_y = \rho Q (-V_2 \sin\theta) - p_2A_2 \sin\theta \quad \ldots(02)$$



The resultant force F acting on the bend is given by $F=\sqrt{F_x^2+F_y^2}$ and the angle made by it with x axis is given by $\tan\alpha=\frac{F_y}{F_x}$

11. A 45° degree bend is connected in a pipe line, the diameters at the inlet and outlet of the bend being 600 mm and 300 mm respectively. Find the force exerted by water on the bend if intensity of pressure at inlet to bend is 88.29 kPa and rate of flow of water is 600 lps. (Jan/Feb 2006)

Solution:

$\theta = 45^0$, $D_1 = 0.6$ m, $D_2 = 0.3$ m

$p_1 = 88.29$ kPa, $Q = 0.6\ m^3/s$

Assume $g = 10\ m/s^2$ and $\rho = 1000\ kg/m^3$

$$A_1=\frac{\pi D_1^2}{4}=\frac{\pi\times 0.6^2}{4}=0.2827\ \text{m}^2$$

$$A_2=\frac{\pi D_2^2}{4}=\frac{\pi\times 0.3^2}{4}=0.07068\,\text{m}^2$$

From discharge continuity equation, we have $Q = A\ V$

$$V_1=\frac{Q}{A_1}=2.122\ \text{m/s and }V_2=\frac{Q}{A_2}=8.488\ \text{m/s}$$

Applying Bernoulli's equation between Sections 1 and 2, we get

$$\frac{p_1}{\rho g}+\frac{\text{v}_1^2}{2\text{g}}+Z_1=\frac{p_2}{\rho g}+\frac{\text{v}_2^2}{2\text{g}}+Z_2$$

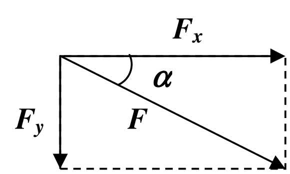
But $Z_1 = Z_2$

Substituting and solving for $p_2 = 54.5$ kPa

$F_x = \rho Q\,[V_1 - V_2\cos\theta] + p_1A_1 - p_2A_2\cos\theta = 19911.4\ N$

$F_y = \rho Q\,[-V_2\sin\theta] - p_2A_2\sin\theta = -6322.2\ N$

$$F=\sqrt{F_x^2+F_y^2}=\sqrt{19911.4^2+6322.2^2}=20891N$$



Acting at $\alpha=\tan^{-1}\left[\frac{F_y}{F_x}\right]=\tan^{-1}\left[\frac{6322.2}{19911.4}\right]=17.616^o$ with horizontal.

July/Aug 2005

12.Water flows up a reducing bend of weight ***80*** kN place in a vertical plane. For the bend, the inlet diameter is ***2*** m, outlet diameter is ***1.3*** m, angle of deflection is ***120°*** and vertical height (distance between the inlet and the outlet) is ***3*** m. If the discharge is ***8.5*** m³/s, pressure at the inlet is ***280*** kPa and the head loss is half the kinetic head at the exit, determine the force on the bend. (12)

Solution:

W = Weight of the reducing bend acting downwards = ***80*** kN (↓), $d_1 = 2$ m,

$d_2 = 1.3$ m, $\theta = 120^o$, $Z = 3$ m, $Q = 2.5$ m^3/s, $p_1 = 280$ kPa. $h_L = 0.5 \frac{V_2^2}{2g}$

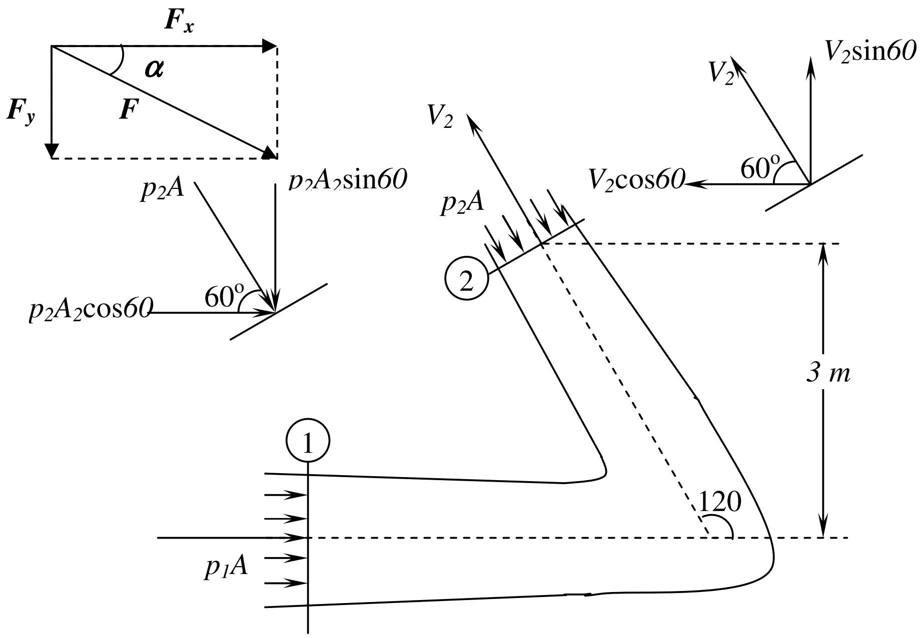
Assume $g = 10$ m/s^2, $\rho = 1000$ kg/m^3, $F_x = ?$ and $F_y = ?$

$A_1 = \frac{\pi d_1^2}{4} = \frac{\pi \times 2^2}{4} = 3.142\,\text{m}^2$ and $A_2 = \frac{\pi d_2^2}{4} = \frac{\pi \times 1.3^2}{4} = 1.327\,\text{m}^2/\text{s}$

Applying discharge continuity equation we have $Q = A_1V_1 = A_2V_2$

Applying Modified Bernoulli's equation between the two sections of the bend shown in Fig. we get

$$\frac{p_1}{\rho g} + \frac{v_1^2}{2g} + Z_1 = \frac{p_2}{\rho g} + \frac{v_2^2}{2g} + Z_2 + h_L$$

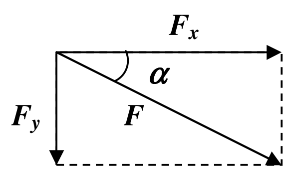


Substituting the values, we get

$$\frac{280 \times 10^3}{1000 \times 10} + \frac{0.7958^2}{2 \times 10} + 0 = \frac{p_2}{1000 \times 10} + \frac{1.8835^2}{2 \times 10} + 3 + 0.5 \times \frac{1.8835^2}{2 \times 10}$$

$p_2 = 247.656$ kPa

Forces acting on the bend in x and y direction respectively are



$F_x = \rho Q[V_1 - V_2 \cos\theta] + p_1A_1 - p_2A_2 \cos\theta = 1{,}048{,}423.63\text{N}$

$F_y = -W + \rho Q[-V_2 \sin\theta] - p_2A_2 \sin\theta = -288{,}688.06 + 80{,}000 = -368{,}688.06\text{N}$

$F = \sqrt{F_x^2 + F_y^2} = \sqrt{1048423.63^2 + 368688.06^2} = 1{,}111{,}360.87$ N

Acting at $\alpha = \tan^{-1}\left[\frac{F_y}{F_x}\right] = \tan^{-1}\left[\frac{288688.06}{1048423.63}\right] = 19.375^o$ with horizontal.

July 2006

In a 45° bend, a rectangular air duct of 1.0 m^2 cross-sectional area is gradually reduced to 0.5 m^2. Find the magnitude of the force required to hold the duct in position, if the velocity of flow is 20.0 m/s at 1 m^2 cross-section and the pressure at both sections is 40 kN/m^2. Specific weight of air is 11.0 N/m^3. (10)

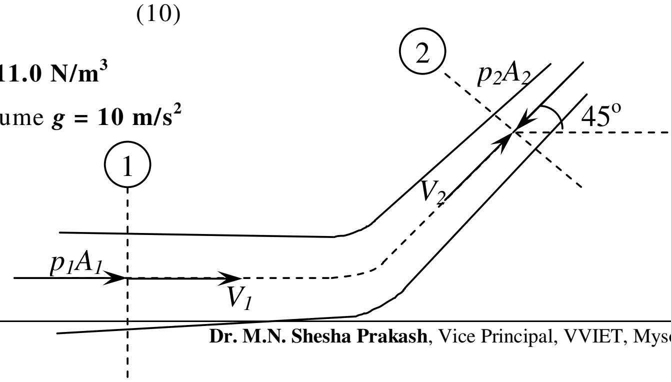
Solution:

$\theta = 45^0$, $A_1 = 1.0$ m^2, $A_2 = 0.5$m^2, $\gamma = 11.0$ N/m^3

$p_1 = p_2 = 40.0$ kPa, $v_1 = 20.0$ m/s, Assume $g = 10$ m/s^2

$\rho = \gamma/g = 1.10$ kg/m^3

From discharge continuity equation,



we have $Q = A_1V_1 = A_2V_2$

$20 = 0.5V_2$

Hence $V_2 = 40$ m/s and

$Q = 20\ m^3/s$

Forces acting on the bend in x and y direction respectively are

$F_x = \rho Q [V_1 - V_2 \cos\theta] + p_1A_1 - p_2A_2 \cos\theta = 25,675.61N$

$F_y = \rho Q [- V_2 \sin\theta] - p_2A_2 \sin\theta = -14,764.39\ N$

$F = \sqrt{F_x^2 + F_y^2} = 29617.97\ N$

Acting at $\alpha = \tan^{-1}\left[\frac{F_y}{F_x}\right] = 29.9^o$ with horizontal