10 CV 35 FLUID MECHANICS NOTES

UNIT-3 Hydrostatic Pressure on Surfaces

by

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3.0 Introduction:

Fluid Static's and Fluid Dynamics form the two constituents of Fluid Mechanics. Fluid Statics deals with fluids at rest while Fluid Dynamics studies fluids in motion. In this chapter we discuss Fluid Static's. A fluid at rest has no shear stress. Consequently, any force developed is only due to normal stresses i.e, pressure. Such a condition is termed the hydrostatic condition. In fact, the analysis of hydrostatic systems is greatly simplified when compared to that for fluids in motion.

Though fluid in motion gives rise to many interesting phenomena, fluid at rest is by no means less important. Its *importance* becomes apparent when we note that the atmosphere around us can be considered to be at rest and so are the oceans. The simple theory developed here finds its application in determining pressures at different levels of atmosphere and in many pressure-measuring devices. Further, the theory is employed to calculate force on submerged objects such as ships, parts of ships and submarines. The other application of the theory is in the calculation of forces on dams and other hydraulic systems.

For a **continuous, hydrostatic, shear free** fluid:

1. Pressure is constant along a horizontal plane,
2. Pressure at a point is independent of orientation,
3. Pressure change in any direction is proportional to the fluid density, local g, and vertical change in depth

Fluid Forces:

In Fluid Mechanics we consider forces upon fluid elements. It is necessary to discuss the type of forces that could act on fluid elements. These Forces could be divided into two categories - Surface Forces, *and* Body Forces.

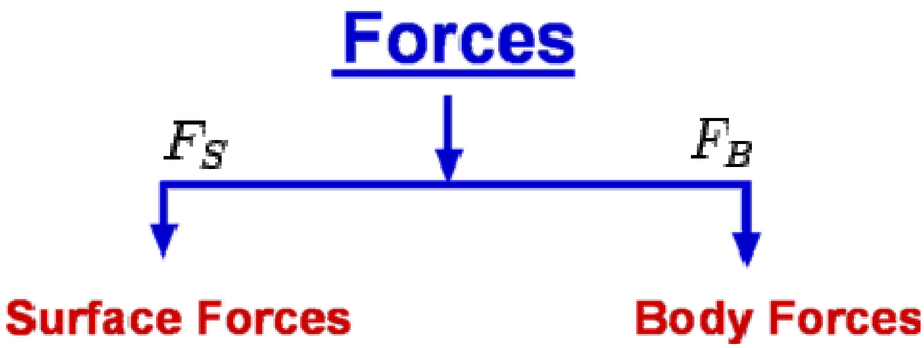


Figure 3.1 : Classification of Fluid Forces

Surface forces are brought about by contact of fluid with another fluid or a solid body. The best example of this is pressure. The surface forces depend upon surface area of contact and do not depend upon the volume of fluid. On the other hand, body forces depend upon the volume of the substance and are distributed through the fluid element. Examples are weight of any substance, electromagnetic forces etc.

Fluid statics is the study of fluids in which there is no relative motion between fluid particles, i.e. no velocity gradient in the fluid. Therefore, no shearing forces exist. Only normal forces exist. These normal forces in fluids are called pressure forces.

3.1 Pressure:

Definition

- Pressure results from normal compressive force acting on an area. Mathematically, it is defined as

$$P = \lim_{\Delta A \to 0} \frac{\Delta F}{\Delta A} = \frac{dF}{dA}$$

Where, *F* is the normal force acting over the area *A*.

- Pressure is a scalar quantity; it has magnitude only and acts equally in all directions.

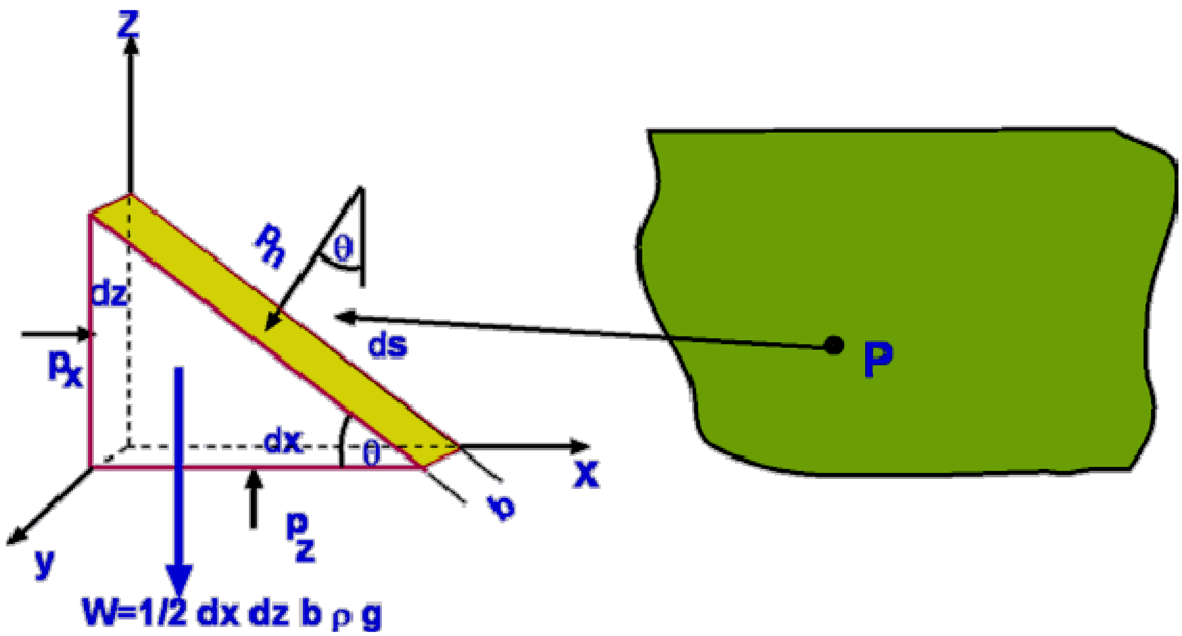


Figure 3.2 : Pressure at a point

The surface forces acting on the three faces of the wedge are due to the pressures, p_x , p_z and p_n as shown. These forces are normal to the surface upon which they act. The usual convention is to consider compression pressure as positive in sign. We again remind ourselves that since the fluid is at rest there is no shear force acting. In addition we have a body force, the weight, 'W' of the fluid within the wedge acting vertically downwards.

$$p_x = p_y = p_z$$

In a closed system, a pressure change produced at one point in the system will be transmitted throughout the entire system. This principle is known as Pascal's law. The Pascal's principle is applied in the development devices like hydraulic brakes, hydraulic jacks and hydraulic lifts.

Absolute Pressure, Gage Pressure and Vacuum:

- A space that is completely evacuated of all gases is called a vacuum.
- The pressure in a vacuum is called *absolute zero* and all pressures referenced with respect to this zero pressure are termed *absolute pressures*.

➢ When pressure is measured relative to the local atmospheric pressure, the pressure reading is called *gage pressure*.

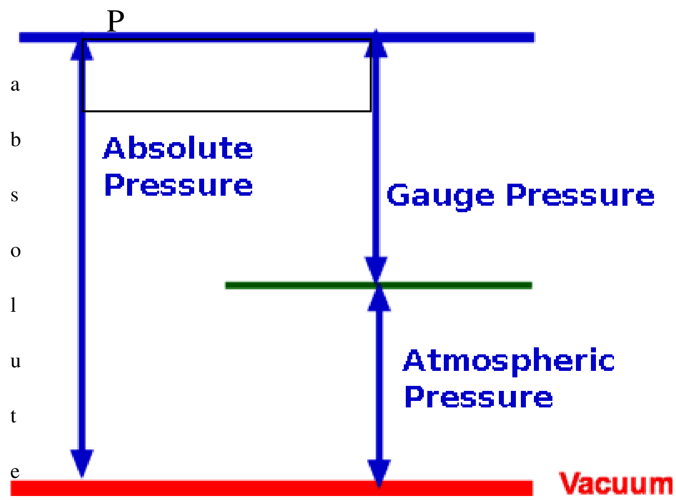


Figure 3.3 : Definition of gauge and absolute pressures

$$P_{abs} = P_{atm} + P_{gauge}$$

➢ When the absolute pressure is less than atmospheric pressure, the gage pressure is negative. Negative gage pressures are also termed *vacuum pressures*, e.g. a gauge pressure of -31.0 kPa can be stated as a vacuum pressure of 31.0 kPa.

➢ The unit of pressure is the Pascal (Pa) in SI system and pounds per square inch (psi) in the traditional system. Gage and absolute pressures are usually identified after the unit, e.g. 50 kPa gauge; 150 kPa absolute;

3.2 Pressure Variation with Elevation:

Basic Differential Equation

➢ For a static fluid, pressure varies only with elevation within the fluid.

➢ Proof: Considering the cylindrical element of fluid shown in fig. 3.4, the following relation can be obtained (proof will be shown in class):

$$\frac{dp}{dz} = \gamma \frac{dz}{dz}$$

Which, can be written as

$$\frac{dp}{dz} = -\gamma$$

This is the basic equation for hydrostatic pressure variation with elevation.

- The following can be observed for a static fluid from the equations above:

- A change of pressure occurs only when there is a change of elevation. Therefore, pressure is constant everywhere in a horizontal plane.
- Pressure changes inversely with elevation.

Pressure Variation for a Uniform-Density Fluid

- For a uniform-density fluid γ is constant. Equation (2) can then be integrated to obtain

$$\underbrace{p + \gamma z}_{piezometric\ pressure} = constant$$

- Dividing equation (3) by γ gives

$$\underbrace{\left(\frac{p}{\gamma} + z\right)}_{piezometric\ head} = constant$$

- Using equations (3) and (4), one can relate the pressure and elevation at two points in a fluid in the following manner:

$$\frac{p_1}{\gamma} + z_1 = \frac{p_2}{\gamma} + z_2$$

Note: The equation applies to two points in the same fluid; it does not apply across an interface of two fluids having different specific weights.

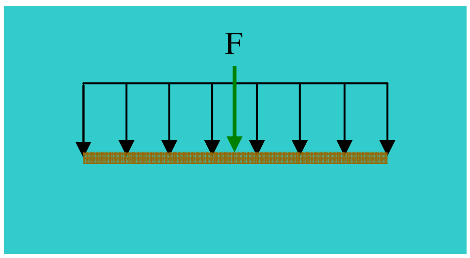
Example Problem

Water occupies the bottom 1.0 m of a cylindrical tank. On top of the water is 0.5 m of kerosene, which is open to the atmosphere. If the temperature is 20^{o}C, what is the gage pressure at the bottom of the tank?

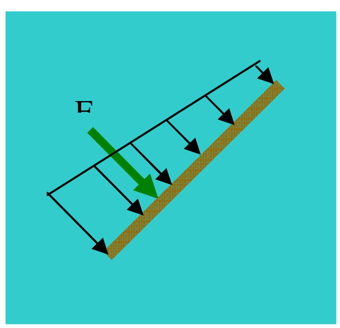
Solution:

3.3 Hydrostatic Forces on Plane Horizontal Surfaces:

- If a plane surface immersed in a fluid is horizontal, then
  - Hydrostatic pressure is uniform over the entire surface.
  - The resultant force acts at the centroid of the plane.



- If a plane surface immersed in a fluid is not horizontal, then
  - Hydrostatic pressure is linearly distributed over the surface.
  - The magnitude and location of the resultant force are obtained by a more general type of analysis.

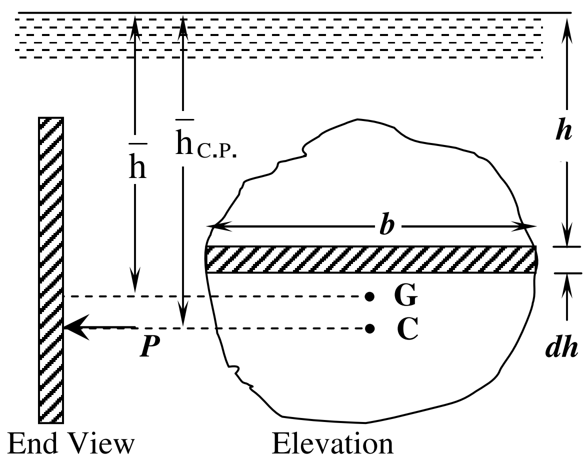


- Consider the plane surface AB immersed in a liquid and inclined at angle α to the liquid surface as shown in figure above.

3.4 Hydrostatic Forces on Vertical Plane Surface:

Vertical Plane surface submerged in liquid

Consider a vertical plane surface of some arbitrary shape immersed in a liquid of mass density ρ as shown in Figure below:



Let, A = Total area of the surface

$\bar{h}$ = Depth of Centroid of the surface from the free surface

G = Centroid of the immersed surface

C = Centre of pressure

$\bar{h}_{C.P.}$ = Depth of centre of pressure

Consider a rectangular strip of breadth b and depth dy at a depth y from the free surface.

Total Pressure:

The pressure intensity at a depth y acting on the strip is p = ρgh

Total pressure force on the strip = dP = (ρgh)dA

∴ The Total pressure force on the entire area is given by integrating the above expression over the entire area $P = \int dP = \int (\rho g h) dA = \rho g \int h\, dA$ Eq.(1)

But ∫ y dA is the Moment of the entire area about the free surface of the liquid given by

$$\int h dA = A\bar{h}$$

Substituting in Eq.(1), we get $P = \rho g A\bar{h} = \gamma A\bar{h}$ Eq.(2)

Where γ is the specific weight of Water,

For water, ρ=1000 kg/m^3 and g = 9.81 m/s^2. The force will be expressed in Newtons (N)

Solved Examples:

Q.1 *A large tank of sea water has a door in the side 1 m square. The top of the door is 5 m below the free surface. The door is hinged on the bottom edge. Calculate the force required at the top to keep it closed. The density of the sea water is 1033 kg/m³.*

Solution: The total hydrostatic force $F = \gamma_{sea \quad water} A \ h_c$

$$\gamma_{sea \quad water} = 1033 \ x 9.81 = 10133.73 \, N/m^3$$

$$\text{Given } A = 1m \text{ X } 1m = 1m^2$$

$$h_c = 5 + \frac{1}{2} = 5.5m$$

$$F = 10133.73 X1X5.5 = 55735.5N$$

Acting at centre of pressure $(y_{c.p})$:

From the above $h_c = 5.5m, \ A = 1m^2$

$$(I_c)_{xx} = \frac{BD^3}{12} = \frac{1X1^3}{12} = 0.08333m^4$$

$$h_{C.P.} = h_c + \frac{(I_c)_{xx}}{Ah_c} = 5.5 + \frac{0.08333}{1X5.5} = 5.515m$$

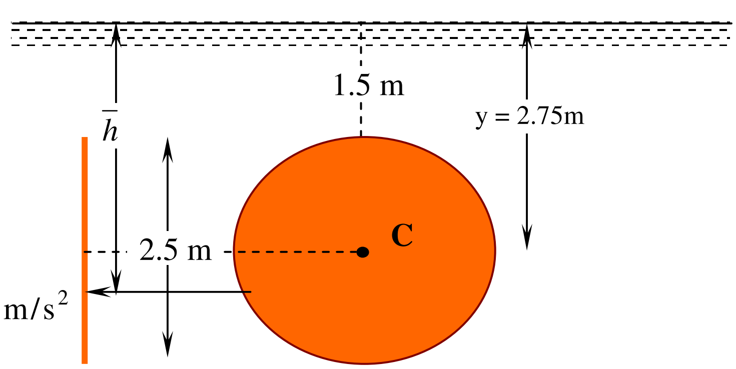
Distance of Hydrostatic force (F) from the bottom of the hinge = 6-5.515 = 0.48485m

The force 'P' required at the top of gate (1m from the hinge)

$$PX1 = FX0.48485 = 55735.5X0.48485$$

$$P = 27023.4 \text{ N} = 27.023 \text{ kN}$$

Q.2 Calculate the total hydrostatic force and location of centre of pressure for a circular plate of 2.5 m diameter immersed vertically in water with its top edge 1.5 m below the oil surface (Sp. Gr.=0.9)



Solution:

$$A = \frac{\pi \times D^2}{4} = \frac{\pi \times 2^2}{4} = 4.91 \ \mathbf{m^2}$$

Assume

$$\rho = 0.9X1000 = 900 \ kg/m^3, g = 9.8 \ m/s^2$$

$$\gamma_{oil} = 900 \text{ X } 9.81 = 8829 \ N/m^3$$

$h_c = 2.75m$

We know that the total pressure force is given by 'F'

$F = \gamma_{oil}\ A\ h_c$ = 8829 x 4.91 x 2.75 = 238184 N = 238.184 kN

Centre of Pressure:

The Centre of pressure is given by

$$h_{C.P.} = h_c + \frac{(I_c)_{x\ x}}{Ah_c}$$

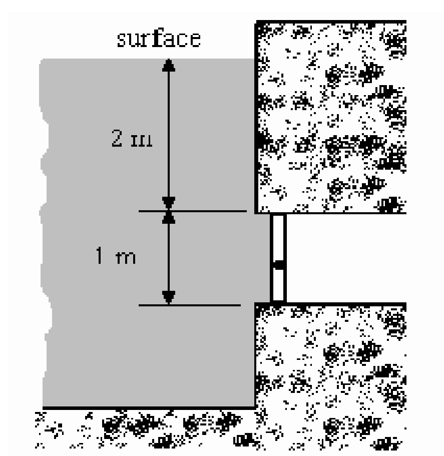
$$I_g = \frac{\pi R^4}{4} = \frac{\pi \times 1.25^4}{4} = 1.9175\ \mathbf{m^4}$$

$$h_{C.P.} = 2.75 + \frac{1.9175}{4.91 \times 2.75} = 2.892\mathbf{m}$$

Q.3 *A culvert in the side of a reservoir is closed by a vertical rectangular gate 2m wide and 1m deep as shown in figure. The gate is hinged about a horizontal axis which passes through the centre of the gate. The free surface of water in the reservoir is 2.5 m above the axis of the hinge. The density of water is 1000 kg/m^3. Assuming that the hinges are frictionless and that the culvert is open to atmosphere, determine*

(i) The force acting on the gate when closed due to the pressure of water.

(ii) The moment to be applied about the hinge axis to open the gate.



Solution: (i) The total hydrostatic force

$$F = \gamma A\ h_c$$

$$\gamma_{water} = 1000\ x9.81 = 9810\ N/m^3$$

Given **A = 1m X 2m = 2m^2**

$$h_c = 2 + \frac{1}{2} = 2.5m$$

$$F = 9810\ X2X2.5 = 49050\ N$$

(ii) The moment applied about hinge axis to open the gate is say 'M'

The centre of pressure ($h_{c.p}$):

From the above $h_c = 2.5m$, $A = 2m^2$

$$(I_c)_{xx} = \frac{BD^3}{12} = \frac{2X1^3}{12} = 0.167m^4$$

$$h_{C.P.} = h_c + \frac{(I_c)_{xx}}{Ah_c} = 2.5 + \frac{0.167}{2X2.5} = 2.53334m$$

Distance of Hydrostatic force (F) from the water surface = 2.5334m.

Distance of hinge from free surface = 2.5m

Distance between hinge and centre of pressure of force 'F' = 2.5334 m - 2.5m = 0.0334m

Taking moment about Hinge to open the gate 'M' = F X 0.0334 = 49050 N X 0.0334 m

The moment applied about hinge axis to open the gate 'M' = 1638.27 N-m

3.4 Hydrostatic Force on a submerged surface:

The other important utility of the hydrostatic equation is in the determination of force acting upon submerged bodies. Among the innumerable applications of this is the force calculation in storage tanks, ships, dams etc.

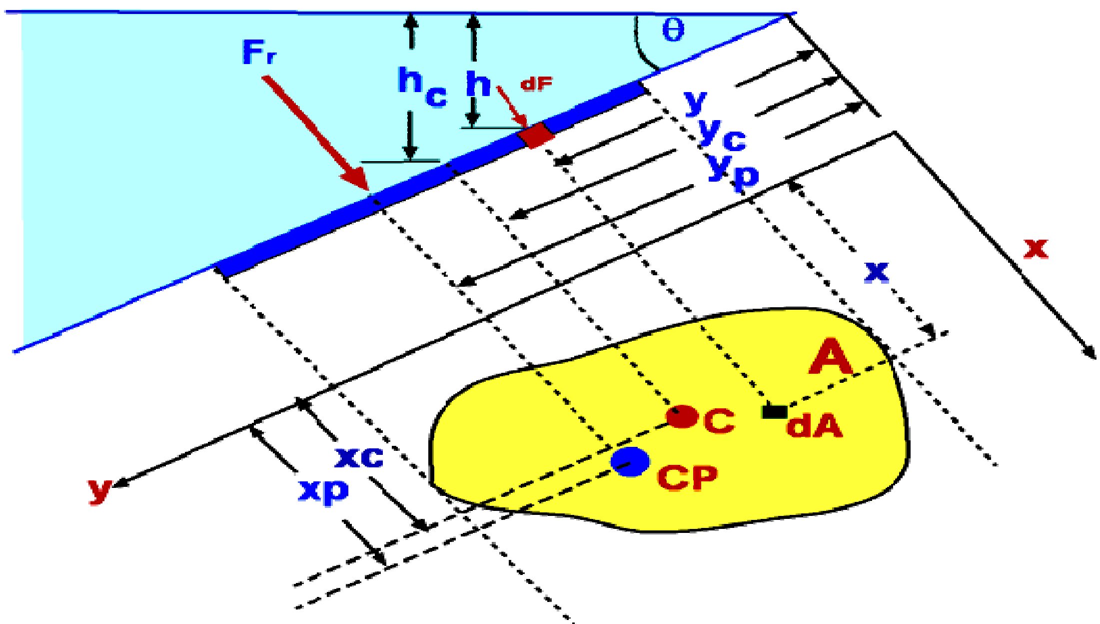


Figure 3.4 : Force upon a submerged object

First consider a planar arbitrary shape submerged in a liquid as shown in the figure. The plane makes an angle θ with the liquid surface, which is a free surface. The depth of water over the plane varies linearly. This configuration is efficiently handled by prescribing a coordinate frame such that the y-axis is aligned with the submerged plane. Consider an infinitesimally small area at a (x,y). Let this small area be located at a depth h from the free surface. dA = dx.dy

Differential Force acting on the differential area dA of plane,

$$dF = (\text{Pressure}) \cdot (\text{Area}) = (\gamma h) \cdot (dA) \qquad \text{(Perpendicular to plane)}$$

Then, Magnitude of total resultant force FR

$$F_R = \int_A \gamma h dA = \int_A \gamma (y \sin\theta) dA$$ Where $h = y\sin\theta$

$$= \gamma \sin\theta \boxed{\int_A y dA}$$ ← 1st moment of the area - Related with the center of area

※ $\int_A y dA = y_c A$ where yc: y coordinate of the center of area (Centroid)

c.f. Center or 1st moment

$$\int_M x dm = MX_C \quad \& \quad \int_M y dm = MY_C$$ (XC & YC: Center of Mass)

$$\int_A x dA = xc \quad \& \quad \int_A y dA = yc$$ (xc & yc: Center of Area)

Moment of inertia or 2nd moment

$$\int_M r^2 dm = I$$ (2nd moment of Mass)

$$\int_A y^2 dA = I_x \quad \& \quad \int_A x^2 dA = I_y$$ (2nd moment of Area)

Then,

$$F_R = \gamma A y_c \sin\theta = (\gamma h_c) A$$

Where γh_c : Pressure at the centroid = (Pressure at the centroid) × Area

- Magnitude of a force on an INCLINED plane
- Dependent on γ, Area, and Depth of centroid
- Perpendicular to the surface (Direction)

i) Position of FR on y-axis 'yR' : y coordinate of the point of action of FR

Moment about x axis:

$$F_R y_R = (\gamma A y_c \sin\theta) y_R = \int_A y dF = \int_A \gamma \sin\theta y^2 dA = \gamma \sin\theta \int_A y^2 dA$$

$$\therefore\ h_R = \frac{\int_A h^2 dA}{h_c A} = \frac{I_x}{h_c A}$$

where $I_x = \int_A y^2 dA$: 2nd moment of area

or, by using the parallel-axis theorem, $I_x = I_{xc} + A y_c^{\,2}$

$$\therefore\ h_{C.P.} = \bar{h} + \frac{I_G \mathrm{Sin}^2\theta^\circ}{A\bar{h}}$$

(The centre of pressure below the centroid)

Worked Examples

Q.4 . A rectangular plate 1.5m x 3.0m is submerged in water and makes an angle of 60° with the horizontal, the 1.5m sides being horizontal. Calculate the magnitude of the force on the plate and the location of the point of application of the force, with reference to the top edge of the plate, when the top edge of the plate is 1.2m below the water surface.

Solution:

$$\bar{h} = \frac{1.2}{\sin 60^\circ} + 1.5 = 1.386 + 1.5 = 2.886\text{m}$$

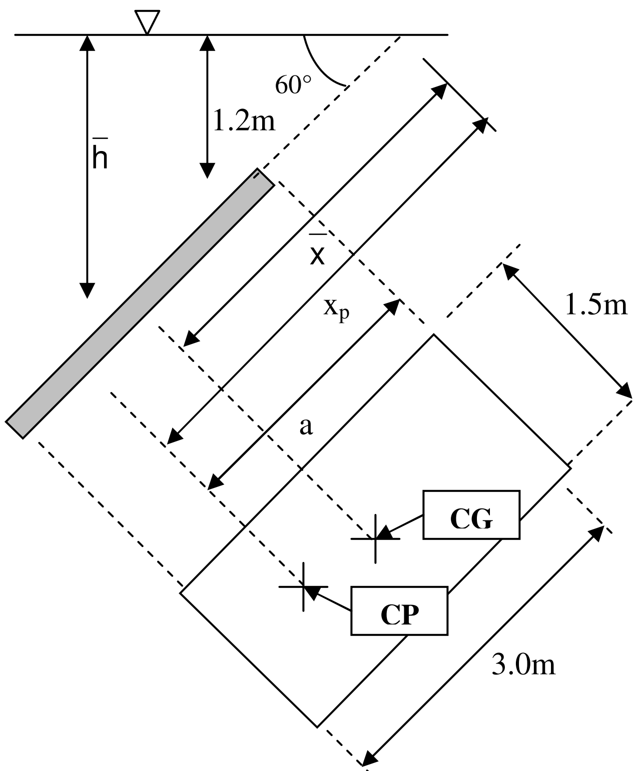
$$A = 3\text{m X } 1.5\text{m} = 4.5\text{m}^2$$

$$\bar{h} = y \sin 60^\circ = 2.886 \sin 60^\circ = 2.499\text{m}$$

$$F = \rho g \bar{h} A = 1000 \times 9.81 \times 2.499 \times 3 \times 1.5$$

$$\therefore \quad F = 109.92 \times 10^3 \text{ N} = 109.92 \text{ kN}$$

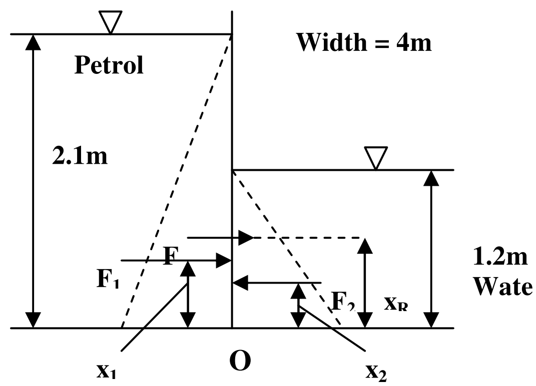
$$h_{C.P.} = \bar{h} + \frac{I_G \sin^2 60^\circ}{A\bar{h}}$$



$$\therefore \quad h_{C.P.} = 2.886 + \frac{3^2}{12 \times 2.886} = 2.886 + 0.260 = 3.146\text{m}$$

From the top edge of the plate, $a = 3.146 - 1.386 = 1.760\text{m}$

Q.5 A vertical bulkhead 4m wide divides a storage tank. On one side of the bulkhead petrol (S.G. = 0.78) is stored to a depth of 2.1m and on the other side water is stored to a depth of 1.2m. Determine the resultant force on the bulkhead and the position where it acts.



Solution:

$$F = \rho g \bar{h} A = \rho g \frac{h}{2} \cdot bh = \tfrac{1}{2}\rho g h^2 \cdot b$$

$$F_1 = \tfrac{1}{2} \times 780 \times 9.81 \times 2.1^2 \times 4 \text{ N} = 67.5 \text{ kN}$$

$$F_2 = \tfrac{1}{2} \times 1000 \times 9.81 \times 1.2^2 \times 4 \text{ N} = 28.25 \text{ kN}$$

Hence the resultant force

$$F_R = F_1 - F_2 = 67.5 - 28.25 = 39.25 \text{ kN}$$

$$h_{C.P.} = \bar{h} + \frac{I_G}{A\bar{h}} = \frac{h}{2} + \frac{bh^3}{12} \quad \frac{1}{bh(h/2)} = \frac{h}{2} + \frac{h}{6} = \tfrac{2}{3}h$$

From the diagram, $y = h - {}^2/_3 h = {}^1/_3 h$

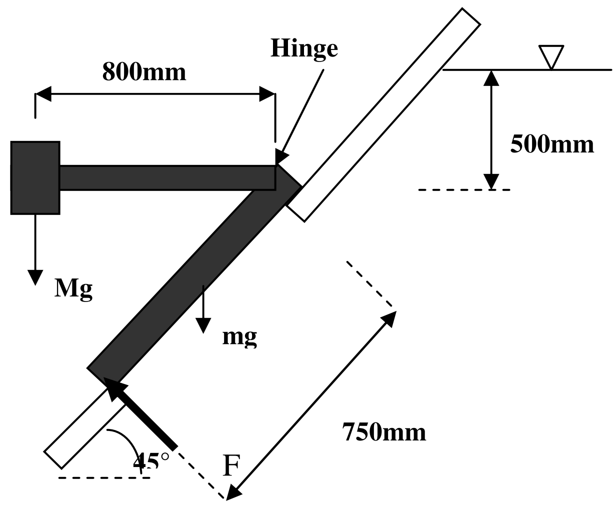
Hence, $y_1 = 2.1 / 3 = 0.7\text{m}$ and $y_2 = 1.2 / 3 = 0.4\text{m}$

Taking moments about 'O', $\boxed{F_R \cdot y_R = F_1 \cdot y_1 - F_2 \cdot y_2}$

i.e. $39.25 \times y_R = 67.5 \times 0.7 - 28.25 \times 0.4$ and hence $y_R = 0.916\text{m}$

Q.6 A hinged, circular gate 750mm in diameter is used to close the opening in a sloping side of a tank, as shown in the diagram in **Error! Reference source not found.**. The gate is kept closed against water pressure partly by its own weight and partly by a weight on the lever arm. Find the mass M required to allow the gate to begin to open when the water level is 500mm above the top of the gate. The mass of the gate is 60 kg. (Neglect the weight of the lever arm.)

Solution:



$$a = \frac{500}{sin\,45°} = 707\text{mm}$$

$$\bar{x} = a + 375 = 1082\text{mm}$$

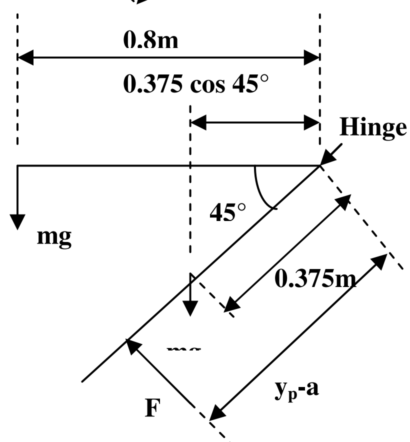
$$\bar{h} = \bar{x}\,sin\,45° = 765\text{mm}$$

$$F = \rho g\bar{h}A = 1000 \times 9.81 \times 0.765 \times \left(\pi \times 0.75^2/4\right)$$

$$\therefore \quad F = 3.315 \times 10^3 \text{ N} = 3.315 \text{ kN}$$

$$x_P = \bar{x} + \frac{I_G}{A\bar{x}} = \bar{x} + \frac{\pi d^4}{64} \cdot \frac{4}{\pi d^2 \bar{x}} = \bar{x} + \frac{d^2}{16 \cdot \bar{x}}$$

$$x_P = 1.082 + \frac{0.75^2}{16 \times 1.082} = 1.082 + 0.032 = 1.114\text{m}$$



Taking moments about the hinge

$$F(x_P - a) = Mg \times 0.8 + mg \times 0.375\,cos\,45°$$

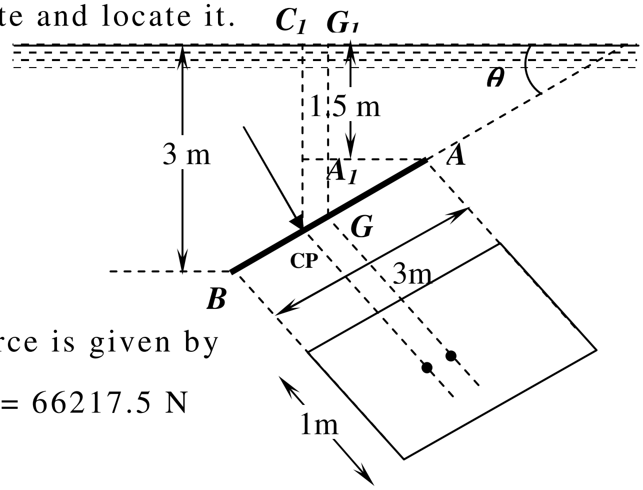
$$3315(1.114 - 0.707) = 9.81(M \times 0.8 + 60 \times 0.375\,cos\,45°)$$

$$M \times 0.8 = \frac{3315(1.114 - 0.707)}{9.81} - 60 \times 0.375\,cos\,45°$$

$$M \text{ X } 0.8 = 137.5 - 16 = 121.5$$

$$\therefore M = \frac{121.5}{0.8} = 152 \text{ kg}$$

Q.7 . A rectangular plate 1 m x 3 m is immersed in water such that its upper and lower edge is at depths 1.5 m and 3 m respectively. Determine the total pressure acting on the plate and locate it.



Solution:

$A = 1 \times 3 = 3\ m^2$

$\gamma_w = 9810\ N/m^3$

$$h_c = \frac{3m + 1.5m}{2} = 2.25m$$

We know that the total pressure force is given by

$F = \gamma_{water} A\ h_c = 9810 \times 3.0 \times 2.25 = 66217.5$ N

Sin θ = 1.5 / 3 = 0.5

θ = 30°

Centre of Pressure

The Centre of pressure is given by

$$(I_C)_{x\ x} = \frac{bd^3}{12} = \frac{1 \times 3^3}{12} = 2.25\,m^4$$

$h_c = 2.25m$

$$CC_1 = h_{C.P.} = h_c + \frac{(I_c)_{x\ x} Sin^2\theta}{Ah_c}$$

$$CC_1 = h_{C.P.} = 2.25 + \frac{2.25\ Sin^2 30^o}{3 X 2.25}$$

$$CC_1 = h_{C.P.} = 2.33333m$$

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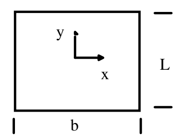
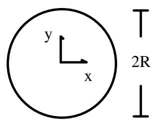
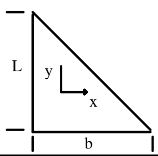
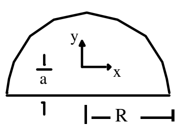
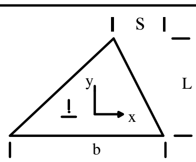
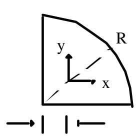
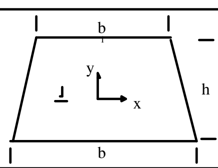
PROPERTIES OF PLANE SECTIONS

Geometry	Centroid	Moment of Inertia I_{xx}	Product of Inertia I_{xy}	Area
	$b/2,\ L/2$	$\frac{bL^3}{12}$	0	$b \cdot L$
	$0, 0$	$\frac{\pi R^4}{4}$	0	πR^2
	$b/3,\ L/3$	$\frac{bL^3}{36}$	$-\frac{b^2L^2}{72}$	$\frac{b \cdot L}{2}$
	$0, a = \frac{4R}{3\pi}$	$R^4\left(\frac{\pi}{8} - \frac{8}{9\pi}\right)$	0	$\frac{\pi R^2}{2}$
	$a = \frac{L}{3}$	$\frac{bL^3}{36}$	$\frac{b(b-2s)L^2}{72}$	$\frac{1}{2} b \cdot L$
	$a = \frac{4R}{3\pi}$	$\left(\frac{\pi}{16} - \frac{4}{9\pi}\right)R^4$	$\left(\frac{1}{8} - \frac{4}{9\pi}\right)R^4$	$\frac{\pi R^2}{4}$
	$a = \frac{h(b+2b_1)}{3(b+b_1)}$	$\frac{h^3(b^2+4bb_1+b_1^2)}{36(b+b_1)}$	0	$(b+b_1)\frac{h}{2}$

Fluid Specific Weight

	1bf/ft^3	N/m^3		1bf/ft^3	N/m^3
Air	.0752	11.8	Seawater	64.0	10,050
Oil	57.3	8,996	Glycerin	78.7	12,360
Water	62.4	9,790	Mercury	846.	133,100
Ethyl	49.2	7,733	Carbon	99.1	15,570

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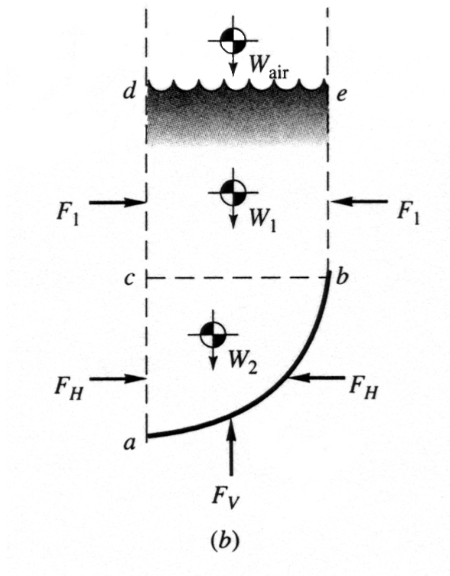
3.5 Hydrostatic Forces on Curved Surfaces

Since this class of surface is curved, the direction of the force is different at each location on the surface.

Therefore, we will evaluate the x and y components of net hydrostatic force separately.



Consider curved surface, a-b. Force balances in x & y directions yield

$$F_h = F_H$$

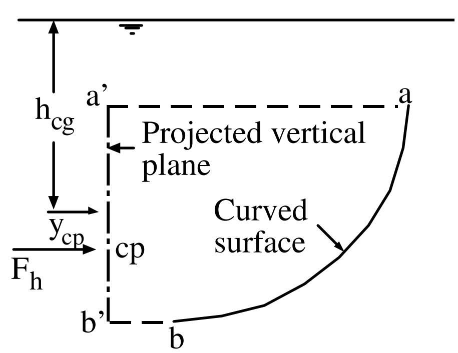
$$F_V = W_{air} + W_1 + W_2$$

From this force balance, the basic rules for determining the horizontal and vertical component of forces on a curved surface in a static fluid can be summarized as follows:

3.5.1 Horizontal Component, F_h

The **horizontal** component of force on a curved surface equals the force on the plane area formed by the **projection of the curved surface** onto a **vertical plane** normal to the component.

The horizontal force will act through the c.p. (**not the centroid**) of the projected area.



Therefore, to determine the horizontal component of force on a curved surface in a hydrostatic fluid:

1. Project the curved surface into the appropriate vertical plane.
2. Perform all further calculations on the vertical plane.
3. Determine the location of the centroid - c.g. of the vertical plane.
4. Determine the depth of the centroid - h_{cg} of the vertical plane.
5. Determine the pressure - $P_{cg} = \rho g h_{cg}$ at the centroid of the vertical plane.
6. Calculate $F_h = P_{cg} A$, where **A is the area of** the projection of the curved surface into the vertical plane, ie. **the area of the vertical plane**.
7. The location of F_h is through the center of pressure of the **vertical plane**, not the centroid.

Note dfrom the Diagram ? All elements of the analysis are performed with the vertical plane. The original curved surface is important only as it is used to define the projected vertical plane.

3.5.2 Vertical Component - F_v

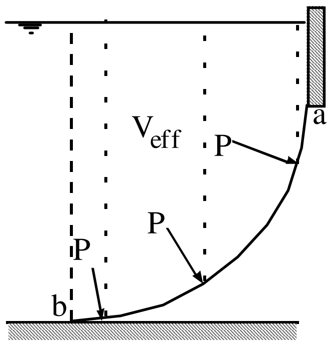
The **vertical** component of force on a curved surface equals the **weight** of the **effective** column of fluid **necessary to cause the pressure on the surface**.

The use of the words **effective column of fluid** is important in that there may not always actually be fluid directly above the surface. (See graphic that follows.)

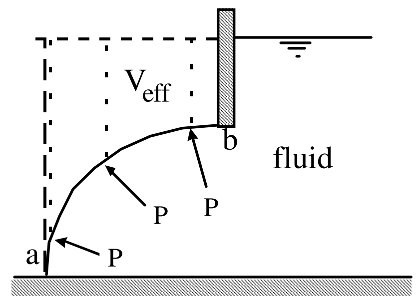
This effective column of fluid is specified by identifying the column of fluid that would be required to cause the pressure at each location on the surface.

Thus, to identify the effective volume - Veff:

1. Identify the curved surface in contact with the fluid.
2. Identify the pressure at each point on the curved surface.
3. Identify the height of fluid required to develop the pressure.
4. These collective heights combine to form V_{eff}.



Fluid above the surface



No fluid actually above surface

These two examples show two typical cases where this concept is used to determine V_{eff}.

The vertical force acts **vertically** through the centroid (center of mass) of the effective column of fluid. The vertical direction will be the direction of the vertical components of the pressure forces.

Therefore, to determine the vertical component of force on a curved surface in a hydrostatic fluid:

1. Identify the effective column of fluid necessary to cause the fluid pressure on the surface.
2. Determine the volume of the effective column of fluid.
3. Calculate the weight of the effective column of fluid - $F_V = \rho g V_{eff}$.
4. The location of F_V is through the centroid of V_{eff}.

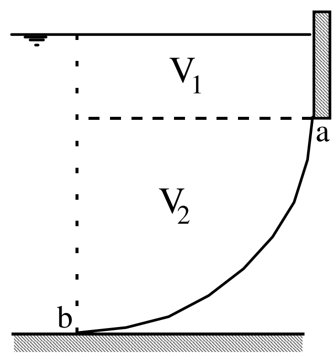
A second problem associated with the topic of curved surfaces is that of finding the location of the centroid of V_{eff}.

Centroid : The location where a point area, volume, or mass can be place to yield the same first moment of the distributed area, volume, or mass, e.g.

$$x_{cg}V_1 = \int_{V_1} x\,dV$$

This principle can also be used to determine the location of the centroid of complex geometries. For example:

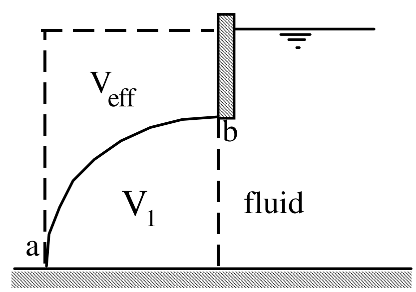
If $V_{eff} = V_1 + V_2$



then

$x_{cg}V_{eff} = x_1V_1 + x_2V_2$

or for the second geometry



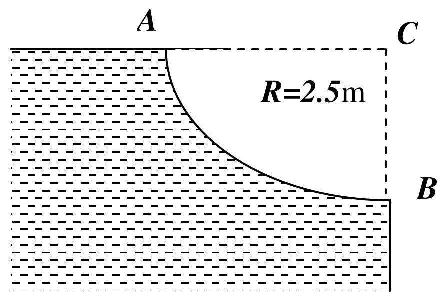
$V_T = V_1 + V_{eff}$

$x_TV_T = x_1V_1 + x_{cg}V_{eff}$

Note: In the figures shown above, each of the x values would be specified relative to a vertical axis through b since the cg of the quarter circle is most easily specified relative to this axis.

Solved Problems:

Ex.1 Find the horizontal and vertical component of force and its point of application due to water per meter length of the gate AB having a quadrant shape of radius 2.5 m shown in Fig. Find also the resultant force in magnitude and direction.



Solution:

Assume

ρ = 1000 kg/m^3 and g = 9.81 m/s^2

R = 2.5 m, Width of gate = 1 m

Horizontal force F_x

F_h = Force on the projected area of the curved surface on the vertical plane

= Force on ***BC***

$A = 2.5 \times 1 = 2.5$ m^2

$$\bar{y} = \frac{2.5}{2} = 1.25\,\text{m}$$

$F = \gamma_{water} A\, h_c = 9810 \times 2.5 \times 1.25 = 30656$ N = 30.656kN

This will act at a distance $\bar{h} = \frac{2}{3} \times 2.5 = \frac{5}{3}\,\text{m}$ from the free surface of liquid AC

Vertical Force F_y

F_y = Weight of water (imaginary) supported by ***AB***

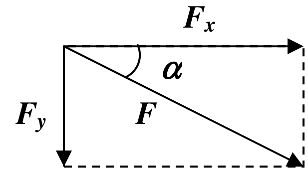
= γ_{water} x Area of ***ACB*** x Length of gate

$$= 9810 \times \frac{\pi \times 2.5^2}{4} \times 1 = 48154N = 48.154kN$$

This will act at a distance $\bar{x} = \frac{4 \times 2.5}{3\pi} = 1.061\,\text{m}$ from ***CB***

The Resultant force

$F = \sqrt{F_x^2 + F_y^2} = \sqrt{30.656^2 + 48.154^2} = 57.084$ ***kN*** and its



inclination is given by

$$\alpha = \tan^{1} \frac{F_y}{F_x} = \tan^{1} \frac{48.154}{30.656} = 57.51^o$$